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A Local Characterization of Darboux(B) Functions. The Semicontinuity of Monotone Functions.

In 1987 Prof. B. Ricceri from Catania (Italy), in a private letter, raised the following problem:

> Let X be a connected, locally connected and complete metric space, and let f be a real-valued function on X such that, for every $r \in \mathbb{R}$, the set $f^{-1}(r)$ is non-empty and arcwise connected. Find some sufficient (and possibly necessary) conditions under which the function f is lower semicontinuous.

Our paper includes a full answer to the question raised by B. Ricceri. At the same time, we discuss important problems of Darboux points, investigated lately very intensely by many mathematicians. Our results are connected with the considerations included in [BB].

The notion of a Darboux point (for real functions of a real variable) was introduced for the first time in paper [BC]. In paper [BB] the authors introduced the notion of real Darboux(B) functions and gave a local characterization of these transformations. Some generalizations of these results are included in [RG]. The consideration of problems of a local characterization of transformations connected with the notion of connectedness can be found, among others, in [JL], [JJ], [GNK], [LS], [RP].

In papers [RG] and [BB] this notion referred to Darboux(B) functions. In our paper the term Darboux(B) will be understood in a bit more general sense than in [RG] and [BB]: Let $f : X \to \mathbb{R}$ where X is some metric space, and let B be a family of connected sets, covering X, (i.e., $X = \bigcup_{A \in B} A$). We say that f is a Darboux(B) function if $f(\bar{U})$ is a connected set for $U \in B$ [1]

Similarly as in [BB] and [RG], as concerns the family B, we shall consider two conditions:

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[1] The difference between our definition and that contained in [RG] and [BB] lies in the fact that, in these papers, B has assumed to be a base of X. In our considerations this assumption is dispensable.