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Positive Linear Functionals on Spaces of Continuous Functions

1. Introduction

In [9] Hausdorff defines a complete ordinary function system Ω on a space X as a linear lattice of continuous functions containing the constants which is uniformly closed, which is a ring, and which is closed under inversion, i.e., if $f \in \Omega$ and $f > 0$, then $1/f \in \Omega$ (here $f > 0$ means that $f(x) > 0$ for all $x \in X$ and $f \geq 0$ means that $f(x) \geq 0$ for all $x \in X$). In particular, each space $C(X)$ of all continuous functions on a topological space is a complete ordinary function system (abbreviated cofs). These systems of functions have been studied by many other authors and we shall refer to some of them in this paper.

If Ω is a cofs, then the bounded functions in Ω form a real Banach algebra under the uniform norm that we shall denote by Ω^*. A representation by measures of the dual space of this Banach space has been obtained by Alexandroff in [1].

The aim of this paper is to represent all positive functionals defined on a cofs Ω by means of integrals. This representation was given by Hewitt in [12], Theorems 13 and 18, when Ω is $C(X)$ for X a realcompact space. Cater in [3] gives a representation of all positive linear functionals defined on $B(X)$, the set of all Baire functions on a realcompact space X, as finite sums of Riesz Homomorphisms. Finally, Tucker in [18] considers a cofs Ω and obtains a representation of all positive linear functionals defined on $B_1(\Omega)$, the set of all pointwise limits of sequences in Ω, as sums of a finite number of Riesz homomorphisms.

2. Preliminaries

$\mathbb{N}$ (resp. $\mathbb{R}, \mathbb{Q}$) will denote the set of all natural numbers (resp. real numbers, rational numbers).

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