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Decreasing Sequences of σ–Fields

Let X be a Polish space, m a Borel probability measure on X, $\{\mathcal{A}_n\}_n$ a decreasing sequence of countably σ–generated σ–fields of Borel sets of X. A martingale $(f_n, \mathcal{A}_n)_n$ is "adapted" to the sequence $\{\mathcal{A}_n\}_n$ if $f^{-1}(\mathcal{B}) = \mathcal{A}_n$ for each n, where $\mathcal{B}$ denotes the family of Borel sets of $\mathbb{R}$. In the simple case in which X is a measuretheoretic product and the $\mathcal{A}_n$'s are the tail–fields, it is easily seen that there always exists an adapted martingale. As a further step to the general case, we settle the case of two fields; this leads to a solution for any *finite* decreasing sequence of fields of the stated type.