**Real Analysis Exchange**
Vol. 18(1), 1992/93, pp. 23–24

Chris Freiling, Department of Mathematics, California State University, San Bernardino, CA 92407

# Totalization: Newton's Problem and Fourier's Problem

We are concerned with the following two problems:

**Problem 1 (Newton's Problem)** *Given a derivative, how can we recover its primitive?*

**Problem 2 (Fourier's Problem)** *Given the limit of an everywhere convergent trigonometric series, how can we recover the series?* The two problems were named by Denjoy, who solved both and considered the solutions to be his two greatest accomplishments. Simpler solutions were discovered by others, and these were harshly criticized by Denjoy for not being "constructive."

The two problems are closely related. Because of theorems of Riemann and Rajchman–Zygmund, respectively, the second problem can be solved by solving either of the following:

**Problem 2a** *Given the Schwarz derivative of a continuous function, how can we recover the function?*

**Problem 2b** *Given the approximate symmetric derivative of a measurable function, how can we recover (almost everywhere) the function?*

There is also a natural intermediate problem:

**Problem $1\frac{1}{2}$** *Given the symmetric derivative of a continuous function, how can we recover the primitive?*

Denjoy's solution to Problem 1 is well known and not too difficult. He calls the method "totalization." His solution to Problem $1\frac{1}{2}$ is much more complicated and involves a finer analysis of perfect sets. Doubts have been raised as to whether this "symmetric totalization" is really constructive. His solution to Problem 2 follows the path 2a and is extremely difficult and confusing.

The goal is to provide new solutions to these problems based upon the "Henstock–Kurzweil" or "Riemann-complete" method. In order to avoid criticism of being "non-constructive" we must first decide what this means. We therefore examine Denjoy's original totalization procedure as it applies to Problem 1. It immediately becomes apparent how the same process works for inverting approximate derivatives and why it runs into trouble for symmetric deriva-