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## Translates of a Set Which Meet It in a Set of Positive Measure

It is well known that the Cantor ternary set  $C$  satisfies  $C + C = [0, 2]$ . One can observe this by considering the set  $C \times C$  and noting that this set meets each line  $x + y = k$  when  $k \in [0, 2]$ . It is also easy to observe from  $C \times C$  that lines  $x + y = k$  which intersect  $C \times C$  in a set of positive  $s'$ -measure ( $s' = \log 2 / \log 3$ ) are those which pass through the corners of squares in the construction of  $C \times C$ ; that is, points  $(x, y)$  where  $x$  and  $y$  are endpoints of intervals contiguous to  $C$ . This implies that there are exactly countably many numbers  $a$  so that  $(C+a) \cap C$  has positive  $s'$ -measure. This yields some curious open questions regarding  $s$ -sets (measurable sets of non-zero finite  $s$ -measure): Given a compact  $s$ -set  $E$  in  $\mathbb{R}^n$  with  $s < n$ , how large can the  $s$ -measure of  $\{t : s\text{-m}((E+t) \cap E) > 0\}$  be? Perhaps it can have positive  $s$ -measure? Perhaps it can be no larger in dimension than  $\lceil s \rceil$ ? If  $E \subset \mathbb{R}^n$  is an  $s$ -set where  $s < n$  is not a whole number, can  $E + E$  be an  $s$ -set?

It is shown in the paper on which this talk is based that any singular,  $\sigma$ -finite, Borel regular measure  $m_a$  whose support is  $E$  (with  $m(E) = 0$  in  $\mathbb{R}^n$ ) satisfies  $m(\{t : m_a((E+t) \cap E) > 0\}) = 0$ . From this result it follows that, if  $E$  is an  $s$ -set or even a set of  $\sigma$ -finite  $s$ -measure in  $\mathbb{R}^n$  with  $s < n$ , then  $m(\{t : s\text{-m}((E+t) \cap E) > 0\}) = 0$ . This fact is then used to show that each  $s$ -set in  $\mathbb{R}^n$  with  $s < n$  is a non-measurable set with respect to any of the approximating measures  $s\text{-}m_\delta$  for any  $\delta > 0$ .