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# The Fractal Analysis of Products and Projections of Measures

Given a Borel measure $\mu$ in $\mathbb{R}^d$, Cutler [1,2] showed that

$$\hat{\mu}(x) = \liminf_{r\downarrow 0} \frac{\log \mu B(x,r)}{\log r}, \quad \hat{\hat{\mu}}(x) = \limsup_{r\downarrow 0} \frac{\log \mu B(x,r)}{\log r}$$

relate directly to the Hausdorff and packing dimensions of measure theoretic supports for $\mu$. We say that $\mu$ is a *fractal measure* if $\hat{\mu}(x) = \hat{\hat{\mu}}(x)$ for $\mu$ a.e. $x$. Using known and new results about the dimension properties of Cartesian products of sets and projections onto subspaces, we find the corresponding results for measures. In particular, if $\mu_1$, $\mu_2$ are Borel measures in $\mathbb{R}$ and $\mu = \mu_1 \times \mu_2$, then $\mu_1$, $\mu_2$ fractal implies that $\mu_1 \times \mu_2$ is fractal. Also, if $\mu_\theta$ denotes the measure in $\mathbb{R}$ obtained by projecting $\mu$ in $\mathbb{R}^2$ onto a straight line of direction $\theta$, then $\mu$ fractal implies that $\mu_\theta$ is fractal for a.e. $\theta$. These results are a corollary to an analysis of the connection between fractal properties of the support sets for $\mu$ and those for $\mu_\theta$; they extend results of Haase [3].

## References

[1] C. D. Cutler, *The Hausdorff dimension distribution of finite measures in Euclidean space*, Can. J. Math. **38** (1986), 1459-1484.

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[3] H. Haase, *On the dimension of product measures*, Mathematika **37** (1990), 316-232.