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GENERALIZED R-COHESIVENESS AND THE ARITHMETICAL HIERARCHY: A CORRECTION TO "GENERALIZED COHESIVENESS"

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Abstract. For $X \subseteq \omega$, let $[X]^n$ denote the class of all n-element subsets of X. An infinite set $A \subseteq \omega$ is called n-r-*cohesive* if for each computable function $f : [\omega]^n \to \{0, 1\}$ there is a finite set F such that f is constant on $[A - F]^n$. We show that for each $n \geq 2$ there is no Π^0_n set $A \subseteq \omega$ which is n-r-cohesive. For $n = 2$ this refutes a result previously claimed by the authors, and for $n \geq 3$ it answers a question raised by the authors.

§1. **Introduction.** A generalized notion of cohesiveness, which arises in connection with effective versions of Ramsey's theorem, was studied by Hummel and Jockusch in [1]. For any set X, let $[X]^n$ denote the class of all n-element subsets of X. A k-*coloring* f of $[X]^n$ is a function $f : [X]^n \to \{0, 1, \dots, k-1\}$. A set $A \subseteq X$ is *homogeneous* for a coloring f of $[X]^n$ if $f \restriction [A]^n$ is constant, i.e., if all n-element subsets of A are assigned the same color by f; n is called the *exponent* of the coloring. An infinite version of Ramsey's theorem states that for any infinite set X and any k-coloring f of $[X]^n$, there exists an infinite set $A \subseteq X$ which is homogeneous for f. A 2-coloring f of $[\omega]^n$ is called *computably enumerable* (or *c.e.*) if either $f^{-1}(0)$ or $f^{-1}(1)$ is c.e. when finite sets are identified with their canonical indices.

DEFINITION 1.1.
(1) A set A is *almost homogeneous* for a coloring f if there exists a finite set F such that $A - F$ is homogeneous for f.
(2) An infinite set $A \subseteq \omega$ is n-*cohesive* (respectively, n-*r-cohesive*) if it is almost homogeneous for every computably enumerable (respectively, computable) 2-coloring of $[\omega]^n$.

It is easy to see that when $n = 1$, we obtain the usual definition of a cohesive or r-cohesive set. Thus, there exists a Π^0_1 1-cohesive set, i.e., a comaximal set (see [5], Theorem X.3.3).

Jockusch [3] (Theorems 4.2 and 5.5) proved that for $n \geq 1$, every computable k-coloring of $[\omega]^n$ has an infinite Π^0_n homogeneous set, and this result was shown to also hold for computably enumerable (c.e.) 2-colorings by Hummel and Jockusch

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