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GENERIC VARIATIONS OF MODELS OF T

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Abstract. Let T be a model-complete theory that eliminates the quantifier $\exists^\infty x$. For T we construct a theory T^+ such that any element in a model of T^+ determines a model of T. We show that T^+ has a model companion T^1. We can iterate the construction. The produced theories are investigated.

§1. Introduction. In [2] Zoé Chatzidakis and Anand Pillay construct generic structures to obtain simple theories. We combine their point of view with an idea in [1] and construct model companions of a new type of theories. But only in few cases we obtain simple theories. Following a suggestion of Lou van den Dries we also use a paper of Peter Winkler [6]. He considers model-complete theories T that eliminate the quantifier $\exists^\infty x$. He shows that certain expansions of T, in particular every Skolem expansion, have a model-completion.

In this paper we also start with a model-complete theory T that eliminates the quantifier $\exists^\infty x$. Let L be the language of T. We replace each n-ary non-logical symbol $R(x_1, \dots, x_n)$, $f(x_1, \dots, x_n)$, or c (if $n = 0$) by a $(n+1)$-ary symbol $R^+(x_0, x_1, \dots, x_n)$, $f^+(x_0, x_1, \dots, x_n)$, or $c^+(x_0)$ respectively. Let L^+ be the new language. Let T^+ be the theory of all L^+-structures M with the following property: If we fix any element a in M, then the relations $R^{+M}(a, x_1, \dots, x_n)$, the functions $f^{+M}(a, x_1, \dots, x_n)$, and the constants $c^{+M}(a)$ determine a model of T on $\operatorname{dom}(M)$. The main result is that T^+ has a model companion T^1 and T^1 again eliminates the quantifier $\exists^\infty x$. Hence we can iterate the construction. We define $T^{n+1} = (T^n)^1$. This is the content of Chapter 2. Before we continue we will give two examples:

1. Let T be the theory of an infinite and coinfinite unary predicate. Then for $n \geq 1$ T^n is the theory of a random $(n+1)$-ary predicate. In this case the T^n are simple theories. They are not stable. Note that the existence of an infinite and coinfinite unary definable predicate in a model of T implies the independence property for T^1.

2. Let T be the theory of an equivalence relation with infinitely many classes and each class infinite. In this case the T^n $(n \geq 1)$ are not simple, as in most of the cases. This example was developed in [1] to investigate the relative strength of the Magidor-Malitz quantifiers Q_1^n where $M \vDash Q_1^n x_1 \dots x_n \varphi(x_1, \dots, x_n)$, if there is an uncountable subset $X \subseteq M$ such that $M \vDash \varphi(a_1, \dots, a_n)$ for all pairwise distinct $a_1, \dots, a_n$ in X. Under the assumption of $\diamondsuit_{\omega_1}$ in [1] for every $n \geq 1$ models M_0 and M_1 of T^n are constructed such that M_0 and M_1 are equivalent with respect to

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