THE JOURNAL OF SYMBOLIC LOGIC
Volume 67, Number 3, Sept. 2002

SOME RESULTS ON PERMUTATION GROUP ISOMORPHISM AND CATEGORICITY

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Abstract. We extend Morley's Theorem to show that if a theory is κ-p-categorical for some uncountable cardinal κ, it is uncountably categorical. We then discuss ω-p-categoricity and provide examples to show that similar extensions for the Baldwin-Lachlan and Lachlan Theorems are not possible.

§1. The basic definitions. We first start by defining a permutation group:

DEFINITION 1.1. A permutation group is a pair $\langle X, G\rangle$ (where X is a set and G is a group) together with an action of G on X such that for any $\sigma \in G$, if for all $x \in X$, $\sigma(x) = x$, then σ is the identity function.

DEFINITION 1.2. Given two permutation groups $\langle X_1, G_1\rangle$ and $\langle X_2, G_2\rangle$, we say that the two permutation groups are isomorphic if there exists a bijection $f : X_1 \to X_2$ such that the map $\sigma \mapsto f\sigma f^{-1}$ is an isomorphism of the groups G_1 and G_2.

DEFINITION 1.3. Given a model M of a theory T, the permutation group associated with the model M is the pair $\langle |M|, Aut(M)\rangle$ (where $|M|$ stands for the universe of the model, and $Aut(M)$ stands for the group of automorphisms of M).

DEFINITION 1.4. Given two models M and N, we will say that M is permutation group isomorphic to N if the permutation group $\langle |M|, Aut(M)\rangle$ is isomorphic to $\langle |N|, Aut(N)\rangle$. Given a theory T, we will say T is κ-p-categorical if all of the models of T of size κ are permutation group isomorphic.

Our goal is to explore whether theorems about categoricity can be extended to p-categoricity. In particular, we will show:

THEOREM 1.5. *Given T, a countable complete theory, if T is κ-p-isomorphic for some uncountable cardinal κ, then T is λ-categorical for all uncountable cardinals.*

We will also provide counterexamples for similar extensions of the Baldwin-Lachlan and Lachlan theorems.

We end this section with two general and needed facts about permutation group isomorphisms. The first has a trivial proof; the second is a simple generalization of the corresponding fact about types (see, for example, Lemma 0.9 in [P]).

Received December 10, 1999; revised August 28, 2001.
1991 *Mathematics Subject Classification.* Primary 03C45.
The authors thank the suggestions of the referee in simplifying several proofs.

0022-4812/02/6703-0002/$1.50