DECONSTRUCTING INNER MODEL THEORY

RALF-DIETER SCHINDLER, JOHN STEEL, AND MARTIN ZEMAN

§1. Introduction. In this paper we shall repair some errors and fill some gaps in the inner model theory of [2]. The problems we shall address affect some quite basic definitions and proofs.

We shall be concerned with *condensation properties* of canonical inner models constructed from coherent sequences \vec{E} of extenders as in [2]. Condensation results have the general form: if x is definable in a certain way over a level $\mathscr{J}_{\alpha}^{\vec{E}}$, then either $x \in J_{\alpha}^{\vec{E}}$, or else from x we can reconstruct $\mathscr{J}_{\alpha}^{\vec{E}}$ in a simple way.

The first condensation property considered in [2] is the *initial segment condition*, or ISC. In section 1 we show that the version of this condition described in [2] is too strong, in that no coherent \vec{E} in which the extenders are indexed in the manner of [2], and which is such that $L[\vec{E}]$ satisfies the mild large cardinal hypothesis that there is a cardinal which is strong past a measurable, can satisfy the full ISC of [2]. It follows that the coherent sequences constructed in [2] do not satisfy the ISC of [2]. We shall describe the weaker ISC which these sequences do satisfy, and indicate the small changes in the arguments of [2] this new condition requires.

In section 2, we fill a gap in the proof that the standard parameters of a sufficiently iterable premouse are solid. This is Theorem 8.1 of [2], one of its central fine structural results. In section 3, we fill a gap in the proof that the Dodd parameter of a sufficiently iterable premouse is Dodd-solid. This is Theorem 3.2 of [4], and is an important ingredient in the proofs of square in $L[\vec{E}]$ and of weak covering for K. The difficulties we overcome in sections 2 and 3 arise from the need to deal with premouse-like structures which do not satisfy even the weaker ISC we introduce in this paper.

In a sense, all of the difficulties we are addressing here stem from the fact that for coherent sequences indexed as in [2], we do not know how to prove that the comparison process terminates without making use of some form of the ISC. Building on an idea of S. Friedman, Jensen has developed the theory of a different sort of coherent sequence. One can think of a Friedman-Jensen sequence as a dilution of a sequence \vec{E} from [2]; it contains the extenders from \vec{E} , interspersed with extenders which only appear on ultrapowers of \vec{E} . Jensen's fine structure theory has many similarities to that of [2], but one way it is significantly simpler is that, granting that there are no extenders of superstrong type on \vec{E} , one can prove a comparison lemma without

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