B. Balcar and F. Franek. *Independent families in complete Boolean algebras*. ***Transactions of the American Mathematical Society***, vol. 274 (1982), pp. 607–618.

Bohuslav Balcar, Jan Pelant, and Petr Simon. *The space of ultrafilters on N covered by nowhere dense sets*. ***Fundamenta mathematicae***, vol. 110 (1980), pp. 11–24.

Boban Velickovic. *OCA and automorphisms of $\mathcal{P}(\omega)/fin$*. ***Topology and its applications***, vol. 49 (1993), pp. 1–13.

The three papers under review, though they treat markedly different subjects, have in common that they mark important points in the developments of these subjects: the Balcar–Franek paper marks an end, the other two mark a beginning.

The main theorem from the Balcar–Franek paper says that a complete Boolean algebra has an independent family of maximum possible size, i.e., if B is complete then it has a subset A of the same cardinality such that

$$\bigwedge F \setminus \bigvee G > 0,$$

whenever F and G are finite disjoint subsets of A. It marks an end because it solves a specific problem and settles a host of questions on complete Boolean algebras (and, dually, extremally disconnected spaces) that depend on the completeness and the cardinality of the algebras alone—some might even argue that it settles all such problems. An instructive example is provided by homomorphic images: if B and C are complete and $|C| \leqq |B|$, then C is a homomorphic image of B; to prove this, map some independent family in B onto C and apply injectivity of C to extend this map to a homomorphism—the dual version states, given two extremally disconnected spaces X and Y, one can embed X into Y iff $w(X) \leqq w(Y)$.

The Balcar–Pelant–Simon paper should have been called *$\mathcal{P}(\omega)/fin$ has a dense tree* (or, for topologists, *ω^* has a tree π-base*) because to many working on this algebra and its Stone space ω^*, this is one of the most important structural facts discovered about these objects. To be precise: there is a dense $2^{\aleph_0}$-branching tree in $\mathcal{P}(\omega)/fin$ whose height equals the distributivity number of the algebra (this cardinal is the same as the minimum cardinality of a family of nowhere dense sets in ω^* with a dense union). This tree has become an important tool in constructions and it is, naturally, useful when one tries to recognize algebras that have the same completion as $\mathcal{P}(\omega)/fin$.

The third paper has in common with the second one that it offers researchers an important handle on the structure of $\mathcal{P}(\omega)/fin$; in this case for dealing with homomorphisms between it and related algebras. In it one finds two things: (1) a proof, building on Veličković's *Definable automorphisms of $\mathcal{P}(\omega)/fin$* (***Proceedings of the American Mathematical Society***, vol. 96 (1986), pp. 130–135), that an automorphism of $\mathcal{P}(\omega)/fin$ with a close-to-definable lifting must be trivial (meaning that it is induced by bijection between cofinite subsets of ω); and (2) a proof from OCA + MA that any automorphism of $\mathcal{P}(\omega)/fin$ must have such a lifting. This can be seen as a synthesis of earlier proofs of the consistency that all automorphisms of $\mathcal{P}(\omega)/fin$ are trivial (Shelah, ***Proper forcing***, JSL L 237, and Shelah and Steprāns, *PFA implies all automorphisms are trivial*, ***Proceedings of the American Mathematical Society***, vol. 104 (1988), pp. 1220–1225).

The methods used in this paper were versatile enough to cause a small flood of results on triviality and non-existence for homomorphisms between $\mathcal{P}(\omega)/fin$ and other algebras; the notes and comments in Farah's ***Analytic quotients, Theory of liftings for quotients over analytic ideals on the integers*** (Memoirs of the American Mathematical Society, no. 702, 2000) give a better overview of these than would be possible in this space.

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