

The second paper proceeds from the existence of certain inner models in which some ordinal countable in  $V$  is a limit of Woodin cardinals. These models are also assumed to be *uniquely iterable* for iteration trees of length up to and including  $\kappa^+$ . This means first of all that these models are iterable, in the sense that the good player wins the corresponding full iteration game of length  $\kappa^+ + 1$ , and furthermore that any corresponding iteration tree of size  $< \kappa^+$  has a unique cofinal branch such that the resulting limit model is itself iterable. The existence of such models has consistency strength below that of infinitely many Woodin cardinals below a measurable cardinal. The proofs here rely on results of Woodin using a construction sometimes (though not here) called the extender algebra; the proofs of these results are given in an eprint version of the paper, posted at an address given in the references. Roughly, these results say that in any sufficiently iterable inner model  $M$ —the models considered here suffice—there is a forcing construction  $P$  in  $M$  such that for any real  $x$  (in any model) there is an iteration  $\pi : M \rightarrow N$  such that  $x$  is (or induces) an  $N$ -generic filter for  $\pi(P)$ . To prove the embedding theorem then one produces for any given real  $z$  an iterate  $M_\infty$  of  $M$  containing  $z$  such that  $\mathbb{R} \cap V$  and  $\mathbb{R} \cap V[G]$  are both the reals of symmetric extensions of  $M_\infty$  by (different) generic filters contained in the same homogeneous forcing, showing that  $L(\mathbb{R})^V$  and  $L(\mathbb{R})^{V[G]}$  satisfy the same sentences with parameters for ordinals and for  $z$ . To show that this symmetric extension works, one needs that the required extender algebras in  $M_\infty$  (really, their power-sets in  $M_\infty$ ) are countable in  $V$ . To ensure that the reals of  $V[G]$  are also a symmetric extension of  $M_\infty$ , an iteration given by a theorem of Woodin is modified to produce a suitable countable version by collapsing a countable elementary submodel (of a sufficiently large initial segment of  $V$ ) in  $V$  whose existence is given by the fact that  $P$  is reasonable. Again, similar, though more involved, methods are used to prove the anticoning theorem.

That iteration trees and the stationary tower can often reproduce one another's results is by now a well-established (though unexplained) phenomenon, although this is perhaps the first time that it has been displayed so explicitly. The authors note as well that the embedding theorem can also be derived from work of Foreman and Magidor, in conjunction with unpublished results of Woodin.

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JAMES CUMMINGS. *A model in which GCH holds at successors but fails at limits*. *Transactions of the American Mathematical Society*, vol. 329 (1992), pp. 1–39.

JAMES CUMMINGS. *Strong ultrapowers and long core models*. *The journal of symbolic logic*, vol. 58 (1993), pp. 240–248.

JAMES CUMMINGS. *Coherent sequences versus Radin sequences*. *Annals of pure and applied logic*, vol. 70 (1994), pp. 223–241.

JAMES CUMMINGS, MATTHEW FOREMAN, and MENACHEM MAGIDOR. *Squares, scales and stationary reflection*. *Journal of mathematical logic*, vol. 1 (2001), pp. 35–98.

The papers under review represent a selection of James Cummings's work in large cardinals and forcing. Throughout,  $\kappa$  and  $\lambda$  will be used to represent uncountable cardinals.

The first of Cummings's papers is related to the singular cardinals hypothesis (SCH), which is one of the most venerable hypotheses in the history of set theory. It can be stated in numerous ways, but for convenience, and in keeping with Cummings's presentation in this paper, we will let SCH be the assertion that if  $\kappa$  is singular, then the size of  $2^\kappa$  is the least cardinal  $\lambda \geq |2^{<\kappa}|$  such that  $\text{cof}(\lambda) > \kappa$ .

Much is known about SCH. A very brief discussion of some of the relevant results will now be given. In the 1970's, Silver showed in his paper *On the singular cardinals problem* (JSL XLVI 864) that if the generalized continuum hypothesis (GCH) fails at a singular