At the bottom levels of the projective hierarchy, deep set-theoretic results had been proved prior to Jackson's work, but these results relied on the theory of indiscernibles for $L(x)$ (x a real), and indiscernibility theory does not generalize to higher levels. So part of Jackson's task was to get new proofs that do not involve indiscernibility. Consider the strong partition property for ω_1, which was first proved by Martin. To prove it, one needs a coding by reals of functions from ω_1 into ω_1 that satisfies a complicated form of the boundedness theorem. AD implies that all such functions are in $L(x)$ for some x, so Martin's coding was simply the real $x^\#$ together with a Skolem term. In the first paper under review, the author gives a new proof by finding a different coding of functions; he analyzes ultrafilters on ω_1 and uses this analysis to get a coding that works. This new proof does generalize, and it constitutes part of the calculation of the projective ordinals.

We have considered projective sets and projective ordinals, but it is natural to go on to more complicated pointsets and larger ordinals. A set of reals is *κ-Suslin* if it is the projection of a tree on $\omega \times \kappa$, and κ is a *Suslin cardinal* if there is a set that is κ-Suslin but not λ-Suslin for any $\lambda < \kappa$. The Suslin cardinals are c.u.b. in their supremum. This concept is the proper generalization of projective ordinals: The first ω Suslin cardinals are the predecessors of the projective ordinals. Thus $\boldsymbol{\delta}^1_{2n+1}$ is a Suslin cardinal and a set is $\boldsymbol{\delta}^1_{2n+1}$-Suslin iff it is $\boldsymbol{\Sigma}^1_{2n+2}$. The generalization of "calculate $\boldsymbol{\delta}^1_n$," calculating a Suslin cardinal κ in terms of the $\aleph$ function, will often be either meaningless or trivial; for example, if κ is sufficiently closed, then $\kappa = \aleph_\kappa$. Nevertheless, it is meaningful to generalize very fine structure up to κ: to analyze ultrafilters and functions on ordinals, to prove partition properties, etc. It seems likely that the methods used for the projective ordinals should handle all the successor Suslin cardinals, and that other known methods will handle singular limit Suslin cardinals. That leaves the case of inaccessible Suslin cardinals.

To see where the problem lies, consider the normal measures on a regular Suslin cardinal, κ. If κ is $\boldsymbol{\delta}^1_{2n+1}$, then for regular $\lambda < \kappa$, the λ-c.u.b. filter on κ is a normal measure, and these are the only normal measures. But if κ is weakly Mahlo, there are stationary sets of regular cardinals, and, as shown by Kleinberg, these stationary sets are associated with normal measures; furthermore, there is a well-ordering of (equivalence classes of) stationary sets similar to the Mahlo order, whose order-type we denote $o(\kappa)$. If $o(\kappa)$ is not too large, the projective ordinal theory generalizes, but if $o(\kappa)$ is too large, there are serious problems which have not yet been overcome. But does there exist a κ such that $o(\kappa)$ is "too large"? In the second paper under review, the author proves that such κ's do exist. This shows that current techniques for developing the very fine structure of $L(\mathbb{R})$ break down at some point (and, in fact, break down at $\kappa^{\mathbb{R}}$, the ordinal of the inductive sets). While today's techniques do not succeed in extending the theory, it is likely that in the future more sophisticated techniques will succeed, and that Jackson's very fine structure theory will extend to all of $L(\mathbb{R})$.

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Itay Neeman and Jindřich Zapletal. *Proper forcings and absoluteness in $L(\mathbb{R})$*. ***Commentationes mathematicae Universitatis Carolinae***, vol. 39 (1998), pp. 281–301.

Itay Neeman and Jindřich Zapletal. *Proper forcing and $L(\mathbb{R})$*. ***The journal of symbolic logic***, vol. 66 (2001), pp. 801–810.

It is by now a standard fact in set theory, due to W. Hugh Woodin, that if there exist certain large cardinals—a proper class of Woodin cardinals suffices—then the theory of the inner model $L(\mathbb{R})$, the smallest model of Zermelo–Fraenkel set theory containing the reals and the ordinals, cannot be changed by set forcing, even if one adds a constant for each real number. The two papers under review extend this result to the theory of $L(\mathbb{R})$ with parameters for real numbers and ordinals, under the restriction that the set forcing in question is *reasonable*.