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model is iterable: instead of simply showing that the unique limit of any linear iteration is well founded, one must provide a strategy that selects, for any iteration tree, a cofinal branch with a well-founded limit (assuming, of course, that this same strategy was used to choose the limit points of the tree).

The definition of the strategy is dealt with in §2 by proving the uniqueness result that, in the absence of a Woodin cardinal, no iteration tree has more than one cofinal branch with a well-founded limit. This result is more important than it may appear, and in fact it is the only result in the paper that is widely cited in its original form.

One application of this basic result appears in §§3–4 of the paper, which use a bookkeeping technique depending on the fact that any initial part $\mathcal{T} \upharpoonright \nu$ of a tree \mathcal{T} has a known well-founded branch, namely the one capped by the ν th node of the tree; hence every other branch of $\mathcal{T} \upharpoonright \nu$ has an ill-founded limit.

Another application, discovered later, provides one way of getting around the fact that the iterability techniques of §§3–4 apply only to countable trees: given an uncountable tree \mathcal{T} , one uses a generic collapse map to obtain a model in which \mathcal{T} is countable and hence has a well-founded branch. The uniqueness theorem of §2 implies that the branch is independent of the collapse map, and hence it must lie in the ground model.

The proof of iterability in §§3–4 is the heart of the paper: §4 presents the basic techniques in illustrative special cases, and the full proof in §4 adds the bookkeeping needed for arbitrary countable trees. The iterability proof given in this paper applies only to the model of §6, but the ideas are much more general and many later papers consist largely of paraphrases of these two sections adapting the proof to more sophisticated models.

The remainder of the paper consists of some open problems in §5, preceding the construction of the promised model in §6. The problems in §5 ask for the elimination of some of the limitations to the techniques of this paper, and later developments have shown that they are overly optimistic: all of the main questions have been answered negatively. Much progress has been made, however, in extending these techniques to better models and to larger cardinals, by using methods that evade or modify some of the limitations. These limitations (the recognition and acceptance of which are partly responsible for the success of this paper) include the assumption that there are no smaller inner models with a Woodin cardinal, in order to use the result of §2; the restriction to countable iteration trees; and the requirement that each extender in the model have a background extender that is a somewhat stronger extender in the real world. It may be that another, comparable breakthrough will be needed to extend this work further in order to construct models containing cardinals in the region of a superstrong cardinal.

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STEVE JACKSON. A new proof of the strong partition relation on ω_1 . Transactions of the American Mathematical Society, vol. 320 (1990), pp. 737–745.

STEVE JACKSON. Admissible Suslin cardinals in $L(\mathbf{R})$. The journal of symbolic logic, vol. 56 (1991), pp. 260–275.

STEVE JACKSON. *A computation of* δ_5^1 . Memoirs of the American Mathematical Society, no. 670. American Mathematical Society, Providence 1999, viii + 94 pp.

In his 1983 UCLA dissertation, Steve Jackson solved the most important open problem in Cabal mathematics, i.e., descriptive set theory under determinacy axioms. The problem was to calculate δ_5^1 . It is easy to explain the precise meaning of "calculating δ_5^1 "; it will be explained, below. But this explanation is misleading, since "calculate δ_5^1 " was, in part, a test question. It was believed that in order to calculate δ_5^1 , one had to develop a very detailed and delicate analysis of all sorts of set-theoretic topics: ultrafilters, sets of ordinals, functions on