a uniform operation u for constructing universes, which assigns a universe uc with $c \in uc$ to every type name c (uniformly in c).

Besides universe construction, EMU includes principles establishing that (i) the underlying structures are always (partial) combinatory algebras; (ii) the collection of types is closed under elementary comprehension and join (a uniform version of disjoint sum); (iii) there exists a "naming relation" $\mathfrak{R}$ correlating objects and types and allowing for a uniform version of the type existence axioms; (iv) universes are closed under elementary comprehension and join.

The main result is a sharp estimate of the proof-theoretic ordinal $|\mathsf{EMU}|$ of EMU. The first step is to establish the lower bound by proving in the theory the well-foundedness of each initial segment of the standard primitive recursive ordering of type $\varphi 1 \varepsilon_0 0$ (φ being a ternary generalization of the Veblen function used for characterizing predicative well-orderings). For proving this theorem, it is crucial to build up a hierarchy of universes for each α below ε_0, and the definition essentially applies the uniform universe constructor u (see the strategy already applied for the theory $\widehat{\mathsf{ID}}_{<\varepsilon_0}$ in Jäger, Kahle, Setzer, and Strahm, *The proof-theoretic analysis of transfinitely iterated fixed point theories*, ***The journal of symbolic logic***, vol. 64 (1999), pp. 53–67).

The upper bound is computed by means of a two-step reduction to $\widehat{\mathsf{ID}}_{<\varepsilon_0}$. EMU-derivations are embedded into a corresponding Tait calculus with ω-rule and then quasi-normalized by partial cut elimination (so that cuts can be reduced to have a suitable Σ-form, i.e., only existential type quantifiers occur, and no reference to the naming relation is made). The preparatory step is followed by a somewhat delicate asymmetric interpretation, which can be formalized by iterated fixed point construction (up to suitably high ordinals below ε_0) in the system $\widehat{\mathsf{ID}}_{<\varepsilon_0}$. Refinements are also possible when restricted forms of number-theoretic induction are present; in particular, it turns out that if number-theoretic induction is restricted to types, then the resulting subsystem $\mathsf{EMU}\restriction$ has the strength of predicative analysis.

To sum up, Strahm's paper is a valuable extension of work by Jäger, et al. (op. cit.); Jäger and Strahm, *Upper bounds for metapredicative Mahlo in explicit mathematics and admissible set theory* (***The journal of symbolic logic***, vol. 66 (2001), pp. 935–958); and Strahm, *Autonomous fixed point progressions and fixed point transfinite recursion* (BSL VII 535). It contributes to the advancement of the project of metapredicative proof theory.

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Toshiyasu Arai. *Consistency proof via pointwise induction*. ***Archive for mathematical logic***, vol. 37 no. 3 (1998), pp. 149–165.

This is a beautiful and concisely written article about a nice application of the Gentzen–Takeuti method (see e.g. Takeuti, ***Proof theory***, JSL LVI 1094, for an exposition) to consistency proofs, which are a classical theme in proof theory. It is well known from Gödel's famous results that theories like PA do not prove their own consistency. Nevertheless if we augment PA by an appropriate transfinite induction scheme PR-TI(ε_0) (for primitive recursive predicates) then the extended system proves $\mathrm{Con}_{\mathrm{PA}}$, and this result can not be improved since PA proves PR-TI(α) for any $\alpha < \varepsilon_0$.

This short and handy description of Gentzen's achievements with respect to ε_0, the proof-theoretic ordinal of PA, is rather vague, and therefore the reader who is interested in a precise and more informative description of what is going on is referred to e.g. Rathjen's recent survey on proof theory and ordinal analysis, *The higher infinite in proof theory*, ***Logic Colloquium '95***, Lecture notes in logic, no. 11, Springer, 1998, pp. 275–304.

In a brief description, the main result of Arai's paper is that over elementary arithmetic (ERA), the scheme of pointwise transfinite induction up to $\psi_0(\varepsilon_{\Omega_{q+1}+1})$, the proof-theoretic