is done about this matter. Many "reductions" of norms to preferences are indicated, for example $Op \longleftrightarrow (\forall s)(\neg p \geqq'^{*} s \rightarrow \neg s \geqq' s)$.

Limitations of situationist deontic logic are overcome first by introducing deontic statements that refer to some situation that does not actually obtain (Chapter 11: *Conflicts and counterfactuals*, hypothetical deontic statements), and secondly by referring to situations and alternative sets in general (Chapters 12 and 13 about *deontic rules*). In the first case, a special implicative connective is introduced (the dyadic conditional operator $O(p/q)$ being judged problematic). Perhaps, Chapter 12 is the most satisfying one. Formalism is here rather clear, normative rules are only sentences of the form $p \Rightarrow \delta$, with $p$ belonging to a descriptive language and $\delta$ belonging to a set of normative sentences like $O\alpha$. There are two kind of rules, unrestrained and restrained: for the latter, "we require that the outcome be logically consistent. We may also want it to be obeyable, which is a stronger requirement than consistency" (p. 193). The analysis can be applied to some elaborate "realistic" cases, which makes it interesting.

Finally, Chapter 13 tries to use the whole formalism for the characterization and classification of legal relations such as rights, claims, and powers. This is a classic, "quasi-obliged" issue in this sort of study, including the comparison with Hohfeld's famous typology. I think that Hansson's study is very clear, and the reader will not be surprised by his conclusion: "As compared to the various typologies, this [Hansson's] framework provides us with greater expressive power, but—as is so often the case—only at the price of loss in terms of simplicity" (p. 222). That judgment properly qualifies the general features of the book. It is a dense and rich study, with an abundant set of formal terms, definitions, and theorems. It is a valuable study also for its numerous examples drawn from real life. Density and richness make it a difficult read— and make an understandable review difficult too.

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WOLFGANG BURR. *Fragments of Heyting arithmetic*. ***The journal of symbolic logic***, vol. 65 (2000), pp. 1223–1240.

During the last quarter of the previous century, a large amount of research has been done on the subsystems of classical first-order Peano arithmetic PA. The progress has been witnessed in a number of monographs, among which the work by P. Hájek and P. Pudlák (***Metamathematics of first-order arithmetic***, JSL LX 1317) is perhaps the most comprehensive one. In comparison, the fragments of Heyting arithmetic HA—the intuitionistic counterpart of PA—largely remain an unexplored territory. The paper under review is an attempt to remedy this situation and to get an insight into this fascinating area.

The main fragments of PA are defined by restricting the formulas in the schemata of induction or collection to the classes $\Pi_n$ or $\Sigma_n$ of the arithmetical hierarchy. A difficulty in the intuitionistic case is that the prenex normal form theorem does not hold and, correspondingly, natural (exhaustive) hierarchies of arithmetical formulas are missing.

A memorable theorem of A. Visser, further improved by K. Wehmeier (*Fragments of HA based on* $\Sigma_1$*-induction*, BSL VII 532), states that the fragment of HA based on the induction schema for all prenex formulas is $\Pi_2$-conservative over the induction schema restricted to $\Pi_2$-formulas only. Thus, intuitionistically, prenex induction is much weaker than full induction, and hence it does not provide a meaningful classification of the fragments of HA. Rather, one would expect that a reasonable classification of arithmetical formulas by their logical complexity should take into account the nestings of implications on a par with the quantifier alternations.

To answer this concern, W. Burr proposes a family of formula classes $\Phi_n$ as the proper intuitionistic analogues of the classes $\Pi_n$ (for $n > 1$). These classes satisfy natural closure