small corrections to the book but more importantly, the current status of many of the open problems in the book can be found there.

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M. HOLZ, K. STEFFENS, and E. WEITZ. ***Introduction to cardinal arithmetic.*** Birkhäuser advanced texts. Birkhäuser Verlag, Basel, Boston, and Berlin, 1999, vii + 304 pp.

The book under review, henceforth referred to as HSW, is the first introductory-level text to have been written since the development of Shelah's pcf theory that includes an introduction to that theory. The authors aim their text at beginners in set theory. They start literally from the axioms and prove everything they need. The result is an extremely useful text and reference book which is also very pleasant to read. Approximately half the text, occupying seven of the nine chapters, is devoted to pcf theory. Pcf theory has had such an impact on the study of cardinal arithmetic that a text on the subject must now cover it. This is a tall order: the theory continues to evolve and is expounded in numerous substantial works of Shelah. Shelah presented some of the theory, as it was up to 1989, in ***Cardinal arithmetic*** (JSL LXII 1035), henceforth referred to simply as ***Cardinal arithmetic***. The pcf theory in HSW does not go beyond what is covered in ***Cardinal arithmetic***. Various people have presented portions of this theory, elaborating on and occasionally simplifying Shelah's arguments, in the form of published or unpublished papers. Much of Chapters 3–7 of HSW is adapted from the dissertation of E. Weitz, *Untersuchungen über die Grundlagen der pcf-Theorie von Saharon Shelah* (Hannover, 1996). The authors have also drawn some of their material on pcf theory from the paper by the reviewer and Menachem Magidor, *Shelah's pcf theory and its applications* (BSL VIII 307) and papers by Thomas Jech, *Singular cardinal problem: Shelah's theorem on* $2^{\aleph_\omega}$ (BSL VIII 308) and *A variation on a theorem of Galvin and Hajnal* (***Bulletin of the London Mathematical Society***, vol. 25 (1993), pp. 97–103). Among the many useful contributions made in HSW toward rendering Shelah's arguments more accessible is an extended discussion, in the final chapter, of Shelah's pp function and its relationship to the Galvin–Hajnal theorems. The first two-thirds of the text (four chapters) contains many exercises of varying levels of difficulty. There are relatively few exercises in the remaining sections on the more advanced pcf theory. The main weakness that the reviewer finds in the book is the scarce commentary on the limitations of the theorems. For example, Chapter 7 deals with sets of regular cardinals a such that $|\mathsf{a}| < \min(\mathsf{a})$ and $\mathrm{pcf}(\mathsf{a})$ has no regular limit points. It does not, however, mention that it is unknown whether the second condition can ever fail in the presence of the first. When it is shown that for any interval of regular cardinals a such that $|\mathsf{a}| < \min(\mathsf{a})$ we have $|\mathrm{pcf}(\mathsf{a})| \leqq |\mathsf{a}|^{+3}$, no comment is made about the unusual exponent.

The recent history of cardinal arithmetic (mainly attempts to prove bounds on cardinal exponentiation) has been well described in this journal by Thomas Jech in the article *Singular cardinals and the pcf theory*, ***Bulletin of symbolic logic***, vol. 1 (1995), pp. 408–424. Very little will be said here about this historical context and the reader is referred to Jech's article for more details. (Note: Theorems 9.5 and 9.6 of that article state more than what is known at present. See the description of Chapters 5–7 below for the correct statements.) It should also be mentioned that Shelah maintains a useful "*Analytical guide*" to his books and papers. A version of it occurs at the end of ***Cardinal arithmetic*** (pp. 435–460) and updates are posted occasionally to Shelah's archive at Rutgers (www.math.rutgers.edu/~shelah/all/E12.pdf).

Let us look more carefully at each of the chapters of the book.

The book assumes little knowledge of set theory. It works in Zermelo–Fraenkel set theory with the axiom of choice (ZFC). A few results are proved without the axiom of choice, and the axiom of choice is shown in ZF to be equivalent to the well-ordering principle and to Zorn's