

FRANK O. WAGNER. *Simple theories*. Mathematics and its applications, vol. 503. Kluwer Academic Publishers, Dordrecht, Boston, and London, 2000, xi + 260 pp.

It is a brave act to write a book in an emerging field. The book under consideration here, *Simple theories* by Frank Wagner, is precisely that. In some sense, it is not accurate to call the study of simple theories an emerging area since it has been emerging for about thirty years. In 1980, Shelah wrote the paper *Simple unstable theories* (*Annals of mathematical logic*, vol. 19 (1980), pp. 177–203) in which the definition of simple theories appears, but in his 1978 book *Classification theory and the number of non-isomorphic models* (JSL XLVII 694), he had already begun the abstract study of dividing and forking. He also introduced a notion of rank called degree or D-rank; if D-rank is ordinal-valued in a theory it is now known as a supersimple theory. Although the next study of simple theories for their own sake after Shelah's *Simple unstable theories* was B. Kim's 1996 dissertation (University of Notre Dame), the generalization of stability theory to unstable settings such as smoothly approximable structures and pseudo-finite fields had progressed throughout the intervening years. Nevertheless, with Kim's proof of symmetry of dividing in simple theories, interest in the field grew enormously, and publishing a book meant to bring the reader to the forefront of research in the area only four years later is quite courageous.

So what is simplicity theory? At its most general, it is the study of the combinatorial notion of dividing, and simple theories are exactly those in which dividing is well behaved. What makes this project somewhat less artificial is the fact that dividing in a simple theory provides a notion of dependence and dimension which, in particular examples, is often best thought of as a generalization of algebraic dependence and dimension from algebraically closed fields. With the success of applications of stability theory in recent years (for instance, Hrushovski, *The Mordell–Lang conjecture for function fields*, JSL LXIII 744), the hope for simple theories lies in the potential for broader applications by keeping a firm handle on some of the motivating examples.

The first chapter of *Simple theories* contains a brief introduction, indicates what a reader must know before venturing further, and has a list of examples. Each chapter, including the first, ends with extensive bibliographic remarks. These are quite helpful for those who wish to delve deeper into a subject or who want to look at the original treatment. The book proper starts in Chapter 2 where the book's main topic, simplicity, is introduced. In any subject, there is a core of fundamentals that anyone who wants to work in an area must know and in the study of simple theories, these fundamentals revolve around the notion of dividing. A type $p(x, B)$ over a small parameter set B inside a saturated model M is said not to divide over some subset A of M if for all A -indiscernible sequences $\langle B_i : i \in I \rangle$ where $\text{tp}(B_i) = \text{tp}(B)$ for all $i \in I$, the conjugates of p , $\{p(x, B_i) : i \in I\}$, are mutually consistent. The author defines a simple theory to be one in which the notion of dividing is symmetric, i.e., $\text{tp}(a/b)$ does not divide over c iff $\text{tp}(b/a)$ does not divide over c . Other equivalent definitions have been used; Shelah's original definition involved the finiteness of certain ranks, the $D(-, \varphi, k)$ -ranks. Requiring that dividing, or a derived notion, forking, have local character, i.e., every type $p(x, B)$ does not divide (fork) over a set $A \subseteq B$ of size at most $|T|$, is also an equivalent definition found in the literature. It seems now that requiring forking to have local character provides the smoothest path through the basics but nonetheless, all the material in Chapter 2 must be internalized by anyone working in the field. Two of the most practical tools are highlighted with their own sections: the independence theorem and a criterion for recognizing when a theory is simple in terms of the presence of an abstract dependence relation. The chapter concludes with a section on stable theories. One of the philosophical positions that the author takes is that stability theory should be seen as a particular instance of simplicity. Although this is literally true—stable theories are particular simple theories—this position hides some of the motivation for many of the constructs in