Its main result can now be easily formulated in a couple of sentences. Let P be any proof system from the following list: Frege, extended Frege, resolution, Horn resolution, the polynomial calculus, the sequent calculus, or the cut-free sequent calculus (this list comprises quite a number of the most popular systems studied in proof complexity). Then, unless $\mathbf{P} = \mathbf{NP}$, there is no polynomial algorithm for approximating $S_P(\varphi)$ *within a multiplicative factor of* $2^{\log^{1-o(1)}|\varphi|}$; in particular, there is no algorithm approximating $S_P(\varphi)$ *within any given constant factor*.

This result is proved by the reduction technique based on the PCP-theorem which by now has become a standard tool in *computational* complexity. Namely, the authors define another optimization problem MMCSA of computing the minimum weight of a satisfying assignment for a given monotone circuit. Based on the PCP theorem, this problem is shown to be hard even to approximate. Then the authors exhibit polynomial reductions from MMCSA to the problem of approximating $S_P(\varphi)$. What I, however, like most in their paper is the fact that it was the first to exhibit the potential of this standard technique in quite a different context of proof complexity. As the following consideration suggests (and also somewhat explains the slow progress in this area), this should not be easy (and was not in the paper under review).

For any reduction of this sort to work, we must have a sufficient stock of both hard and easy tautologies φ to use in the image of the reduction. Whereas it is generally not a problem with easy ones, possessing hard tautologies should also include a *proof* that they are really hard, and our current abilities for proving lower bounds on $S_P(\varphi)$ (especially not for a single tautology, but for a large class of them, as required by the reduction) are rather limited. This is the major difficulty the authors had to overcome, and they indeed introduced quite a few novel and interesting techniques for this purpose.

As a conclusion, let me mention that it is a widespread belief that for sufficiently strong propositional proof systems, efficient search algorithms which perform reasonably well for *all* tautologies do not exist. The paper under review makes a new significant step toward confirming this belief and paves the way for further progress in this direction.

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LEV D. BEKLEMISHEV. *Induction rules, reflection principles, and provably recursive functions*. ***Annals of pure and applied logic***, vol. 85 (1997), pp. 193–242.

According to a classical result by Kreisel and Lévy (*Reflection principles and their use for establishing the complexity of axiomatic systems*, JSL XXXVI 529), the uniform reflection principle for EA (also known as $I\Delta_0 + EXP$) and the schema of induction are equivalent over EA. Therefore the *rule* of induction and the uniform reflection principle for EA are also equivalent, simply because the rule of induction and the schema of induction are equivalent.

Now one can ask how much reflection is needed for how much induction. Leivant (*The optimality of induction as an axiomatization of arithmetic*, ***The journal of symbolic logic***, vol. 48 (1983), pp. 182–184) has refined Kreisel's and Lévy's result by investigating restricted schemata of induction; he proved that $I\Sigma_k$ is equivalent to EA plus uniform reflection for Σ_{k+1}-formulas, if $k \geqq 1$.

Beklemishev takes up the task of relating reflection schemata and restricted induction *rules*. He considers not only restrictions on the complexity of the formulas allowed in the rule of induction but also restrictions on the number of allowed iterations of the rules. In particular, he considers for theories T and rules R the closure $[T, R]$ of T under uniterated applications of R. That is, a proof in $[T, R]$ may contain several applications of R, but they are not to occur on the same branch within the proof.