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REVIEWS

The Association for Symbolic Logic publishes analytical reviews of selected books and articles in the field of symbolic logic. The reviews were published in ***The Journal of Symbolic Logic*** from the founding of the JOURNAL in 1936 until the end of 1999. The Association moved the reviews to this BULLETIN, beginning in 2000.

The Reviews Section is edited by Alasdair Urquhart (Managing Editor), Steve Awodey, John Baldwin, Lev Beklemishev, Mirna Džamonja, David Evans, Erich Grädel, Denis Hirschfeldt, Roger Maddux, Luke Ong, Grigori Mints, Volker Peckhaus, and Sławomir Solecki. Authors and publishers are requested to send, for review, copies of books to *ASL, Box 742, Vassar College, 124 Raymond Avenue, Poughkeepsie, NY 12604, USA.*

In a review, a reference "JSL XLIII 148," for example, refers either to the publication reviewed on page 148 of volume 43 of the JOURNAL, or to the review itself (which contains full bibliographical information for the reviewed publication). Analogously, a reference "BSL VII 376" refers to the review beginning on page 376 in volume 7 of this BULLETIN, or to the publication there reviewed. "JSL LV 347" refers to one of the reviews or one of the publications reviewed or listed on page 347 of volume 55 of the JOURNAL, with reliance on the context to show which one is meant. The reference "JSL LIII 318(3)" is to the third item on page 318 of volume 53 of the JOURNAL, that is, to van Heijenoort's *Frege and vagueness*, and "JSL LX 684(8)" refers to the eighth item on page 684 of volume 60 of the JOURNAL, that is, to Tarski's *Truth and proof*.

References such as 49***5*** or 280***1*** are to entries so numbered in *A bibliography of symbolic logic* (the JOURNAL, vol. 1, pp. 121–218).

ILIJAS FARAH. ***Analytic quotients.*** Memoirs of the American Mathematical Society vol. 148 no. 702, American Mathematical Society, Providence, R.I., 2000, xvi + 177 pp.

The Boolean algebra $\mathcal{P}\mathbb{N}$ of subsets of the natural numbers has some very well-known ideals, starting with the ideal $[\mathbb{N}]^{<\omega}$ of finite sets and the ideal $\mathcal{Z}$ of sets of asymptotic density zero; going a little farther we have the ideal $\mathcal{Z}_{\log}$ of sets $A \subseteq \mathbb{N}$ such that

$$\lim_{n\to\infty} \frac{1}{\ln n} \sum_{i\in A\cap n} \frac{1}{i+1} = 0.$$

For any such ideal we can consider the corresponding quotient Boolean algebra $\mathcal{P}\mathbb{N}/\mathcal{I}$. The algebra $\mathcal{P}\mathbb{N}/[\mathbb{N}]^{<\omega}$ has long been recognised as one of the fundamental objects of set-theoretic analysis, topology and combinatorics. The others have not been studied so systematically, but show on the briefest of acquaintanceships the potential for generating fascinating questions. In this extraordinary monograph we are given some tools for tackling these questions which are surely going to be part of the essential kit for anyone working in the area. The most striking results concern the representation of Boolean homomorphisms between quotient algebras in terms of functions from $\mathcal{P}\mathbb{N}$ to itself or between cofinite subsets of $\mathbb{N}$.

The first steps are already far from being obvious; we need definitions which will lead to a useful classification scheme. One is well known. A *P-ideal* is an ideal $\mathcal{I}$ such that for every sequence $\langle I_n\rangle_{n\in\mathbb{N}}$ in $\mathcal{I}$ there is an $I \in \mathcal{I}$ such that $I_n \setminus I$ is finite for every n. The next is natural enough to the lateral thinker: an ideal $\mathcal{I}$ is *analytic* if it is an analytic

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