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CORRIGENDUM TO: "QUANTIFIER ELIMINATION IN VALUED ORE MODULES"

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Recall that $A := K[t;\sigma,\partial]$ is a skew polynomial ring. Lemma 2.4 in [1] should read as follows (but note there is no effect on the rest of the paper):

LEMMA 2.4. For any pair $\{q_1(t), q_2(t)\}$ of elements of A, we have the following equivalence in any divisible A-module M:

$$ann_M(q_2(t)) \subseteq ann_M(q_1(t)) \text{ if and only if}$$
$$\text{there exists } q_3(t) \text{ such that } (ann_M(q_2(t)) = ann_M(q_3(t)) \text{ and } q_3(t) \text{ divides } q_1(t)).$$

Moreover, if $q_1(t) = q_2(t) \cdot r(t)$ and if the cardinality $|ann_M(q_1(t))/ann_M(q_2(t))|$ is finite, then $|ann_M(q_1(t))/ann_M(q_2(t))| = |ann_M(r(t))|$.

For the convenience of the reader we give a proof below (of the first part), along the lines of Lemma 2.9 and Proposition 2.10 of reference [17] as indicated in [1]. The argument also shows the following: for any pair of elements $\{q_1(t), q_2(t)\}$ of A, we have that $q_2(t)$ divides $q_1(t)$, whenever $ann_M(q_2(t)) \subseteq ann_M(q_1(t))$, $deg(q_1(t)) > deg(q_2(t))$, and $ann_M(q(t)) \neq \{0\}$ for any $q(t) \notin K$ which divides $q_2(t)$ on the right.

PROOF OF THE LEMMA. We will proceed by induction on the sum of the degrees of $q_1(t)$ and $q_2(t)$, assuming that both $q_1(t)$, $q_2(t)$ are non-zero. Either $ann_M(q_2(t)) = \{0\}$, then take $q_3(t) = 1$, or $ann_M(q_2(t)) \neq \{0\}$.

So let $0 \neq u \in ann_M(q_2(t))$ and let $q(t) \in A - \{0\}$ with minimal degree such that $u \cdot q(t) = 0$. Note that $deg(q(t)) \geq 1$. Applying the right Euclidean algorithm, we have that $q_2(t) = q(t) \cdot r_2(t)$ and since $ann_M(q_2(t)) \subseteq ann_M(q_1(t))$, that $q_1(t) = q(t) \cdot r_1(t)$ for some $r_1(t), r_2(t) \in A - \{0\}$.

Let us show that $ann_M(r_2(t)) \subseteq ann_M(r_1(t))$. Let $u' \in ann_M(r_2(t))$. Since M is divisible, there exists u'' such that $u'' \cdot q(t) = u'$. So $u'' \in ann_M(q_2(t)) \subseteq ann_M(q_1(t))$ and so $0 = u'' \cdot q(t) \cdot r_1(t) = u' \cdot r_1(t)$. So we may apply induction to the pair $(r_1(t), r_2(t))$ since $deg(r_1(t)) + deg(r_2(t)) < deg(q_1(t)) + deg(q_2(t))$. Therefore, there exists $r_3(t)$ with $ann_M(r_3(t)) = ann_M(r_2(t))$ and $r_3(t)$ divides $r_1(t)$. It remains to note that $ann_M(q(t) \cdot r_3(t)) = ann_M(q_2(t))$. So let $u \in ann_M(q(t) \cdot r_3(t))$, then $u \cdot q(t) \in ann_M(r_2(t))$ and so $u \in ann_M(q_2(t))$. Conversely let $u \in ann_M(q_2(t))$, so $u \cdot q(t) \in ann_M(r_2(t))$. Since $ann_M(r_2(t)) = ann_M(r_3(t))$, we have $u \cdot q(t) \cdot r_3(t) = 0$. $\dashv$

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