THE JOURNAL OF SYMBOLIC LOGIC
Volume 76, Number 3, Sept. 2011

RETRACTION OF: "A NORMALIZATION THEOREM FOR SET THEORY"

SIDNEY C. BAILIN

The paper *A normalization theorem for set theory*, this JOURNAL, vol. 53 (1988), no. 3, pp. 673–695, contains an error that invalidates the purported theorem. The paper modifies the usual natural deduction formalism of Zermelo (or Zermelo–Fraenkel) set theory by requiring the immediate inference of $\perp$ from premises of the form $t \in t$, with the intention of circumventing M. Crabbé's counterexample to normalization in set theory, in which t is $\{x \mid x \in a \wedge (x \in x \to \perp)\}$ [2]. There are, however, generalizations of Crabbé's counterexample that are not covered by these inferences: given any formula A that is equivalent modulo rewriting to $A \to B$ for some B, a derivation of A can be applied, in effect, to itself. The resulting self-application, when applied to itself, yields a non-normalizable derivation.

The error in the paper appears to be in Lemma 4.10, in the case $B = y \in S$, which is supposedly reduced to the case $z \in S$ for a new variable z; the latter case has not, however, been proven at that point.

Inferring $\perp$ immediately from $t \in t$ where t is $\{x \mid x \in a \wedge (x \in x \to \perp)\}$ may be viewed as a special case of *deduction modulo*, which is defined and developed in a more general manner by Dowek and Werner [1]. Although my paper is cited in [1], they do not build on the claimed result. My paper should not be cited in any future work on this topic.

REFERENCES

[1] G. DOWEK and B. WERNER, *Proof normalization modulo*, this JOURNAL, vol. 68 (2003), no. 4, pp. 1289–1316.

[2] L. HALLNÄS, ***On normalization of proofs in set theory***, Ph.D. thesis, University of Stockholm, Stockholm, 1983.

Received March 10, 2011.

1943-5886/11/7603-0018/$1.10