THE JOURNAL OF SYMBOLIC LOGIC
Volume 74, Number 4, Dec. 2009

CORRIGENDUM TO: "ON THE STRENGTH OF RAMSEY'S THEOREM FOR PAIRS"

PETER A. CHOLAK, CARL G. JOCKUSCH, JR., AND THEODORE A. SLAMAN

Several proofs given in [2] contain significant errors or gaps, although to our knowledge all results claimed there are provable. The needed corrections are described below. All references are to [2] unless otherwise stated, and we adopt the notation and terminology of that paper.

1. Lemma 7.10 asserts that the principles D^2_2 and SRT^2_2 are equivalent over RCA_0. However, the proof that D^2_2 implies SRT^2_2 has a hidden application of $\mathsf{B}\Sigma^0_2$ and thus is actually carried out in $\mathsf{RCA}_0 + \mathsf{B}\Sigma^0_2$. The problem is that, in the construction of H by adding one element at a time, each element c added to H must form a pair of the appropriate color with all previously chosen elements. To get the existence of such a c one seems to need $\mathsf{B}\Sigma^0_2$. This gap was recently closed by Chong, Lempp, and Yang, who showed in [3], Theorem 1.4, that, in RCA_0, D^2_2 implies $\mathsf{B}\Sigma^0_2$, and hence D^2_2 implies SRT^2_2.

2. Lemma 7.11 asserts that RT^2_2 is equivalent to SRT^2_2 & COH over RCA_0. However, the proof given there that RT^2_2 implies COH in RCA_0 is seriously flawed. This was pointed out by Joseph Mileti and later by Jeffrey Hirst. A proof that RT^2_2 implies COH in $\mathsf{RCA}_0 + \mathsf{I}\Sigma_2$ can easily be extracted from the proof of Theorem 12.5. Mileti, and simultaneously Lempp and Jockusch, observed that it is possible to eliminate the use of $\mathsf{I}\Sigma_2$ by effectively bounding in terms of k the number of changes in the characteristic function of A when it is restricted to A_k, so that proving that this number is finite requires only Σ_1-induction. Thus, it is provable in RCA_0 that RT^2_2 implies COH, and hence that RT^2_2 is equivalent to SRT^2_2 & COH.

3. Joseph Mileti pointed out a gap in the proof of the claim at the bottom of page 50 that a certain computable 2-coloring of pairs C is "jump universal" in the sense that for every C-homogeneous set A and every computable coloring $\tilde{C}$, there exists an infinite $\tilde{C}$-homogeneous set B with $B' \leq_T A'$. The proof provided works only when $\tilde{C}$ is stable. However, this assumption can be eliminated by using the density of the Turing degrees under $\ll$ (see [6], Theorem 6.5) to stabilize $\tilde{C}$. Namely, by Theorem 12.5 let C be a computable coloring such that every infinite homogeneous set has jump of degree $\gg \mathbf{0}'$, and let A be an infinite homogeneous set for C. Let $\mathbf{d}$ be the degree of A', so that $\mathbf{d} \gg \mathbf{0}'$. Let $\tilde{C}$ be any computable 2-coloring of pairs. We must show that $\tilde{C}$ has an infinite homogeneous set with jump of degree at most $\mathbf{d}$. Let $\mathbf{c}$ be a degree with $\mathbf{d} \gg \mathbf{c} \gg \mathbf{0}'$, and let $\mathbf{a}$ be a degree with $\mathbf{a}' = \mathbf{c}$. Since

Received June 30, 2009.
2000 *Mathematics Subject Classification.* Primary: 03F35 03C62 03D80.