

CORRIGENDUM TO:
“ON LASCAR RANK AND MORLEY RANK OF DEFINABLE GROUPS
IN DIFFERENTIALLY CLOSED FIELDS”

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In [3], we proved

THEOREM. *Let T be an ω -stable theory of Morley rank ω^α . Suppose for any ordinal $1 \leq \tau \leq \alpha$ and integer $d \geq 1$, T^{eq} satisfies the condition:*

$$\text{For any complete type } p, \text{ if } \text{RM}(p) = \omega^\tau d \text{ then } \text{RU}(p) = \text{RM}(p). \quad (*)$$

Then Morley rank and U-rank agree for definable groups in T^{eq} .

We then claimed [3, Corollary 1.4] that $m\text{-DCF}_0$, the theory of characteristic 0 differentially closed fields with m commuting derivations, is an ω -stable theory satisfying (*). However, our proof is not valid. The fallacy is that in justifying (*) for $m\text{-DCF}_0$, we used in an essential way the following result¹ of McGrail [2, Theorem 5.2.2]

If p is a complete type over a model of $m\text{-DCF}_0$ with Δ -type τ and typical Δ -dimension a_τ , then $\omega^\tau a_\tau \leq \text{RU}(p)$.

which was shown to be false by Sonat Süer[5]. He constructed, for any $k \geq 1$, U-rank ω types in $m\text{-DCF}_0$ ($m \geq 2$) with Δ -type 1 and typical Δ -dimension k . Süer’s counter-examples will appear in his forthcoming paper.

At this point, we would like to make two more remarks:

- Our proofs remain valid in the single derivation case. The second author showed that (Theorem 2.7 [4]) in DCF_0 , if the Morley rank of a type is a limit ordinal then its Morley rank and U-rank coincide.
- Currently, we are not able to decide, for $m \geq 2$, whether $m\text{-DCF}_0$ satisfies (*) or not. The examples given by Süer all have their Morley ranks and U-ranks equal. Hence they do not show that $m\text{-DCF}_0$ fails (*).

REFERENCES

- [1] E. R. KOLCHIN, *Differential algebra and algebraic groups*, Pure and Applied Mathematics, vol. 54, Academic Press, New York–London, 1973.
[2] T. MCGRAIL, *The model theory of differentially closed fields with several commuting derivations*, this JOURNAL, vol. 65 (2000), no. 2, pp. 885–913.

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¹The terminologies here are slightly different from those in [2]. See Remark 4.2.3 in [2] and pp. 129–130 in [1] for the translation between them.