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RECONSTRUCTION OF HOMOGENEOUS RELATIONAL STRUCTURES

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§1. Introduction. This paper contains a result on the reconstruction of certain homogeneous transitive ω-categorical structures from their automorphism group. The structures treated are relational. In the proof it is shown that their automorphism group contains a *generic pair* (in a slightly non-standard sense, coming from Baire category).

Reconstruction results give conditions under which the abstract group structure of the automorphism group $\mathrm{Aut}(\mathcal{M})$ of an ω-categorical structure $\mathcal{M}$ determines the topology on $\mathrm{Aut}(\mathcal{M})$, and hence determines $\mathcal{M}$ up to bi-interpretability, by [1]; they can also give conditions under which the abstract group $\mathrm{Aut}(\mathcal{M})$ determines the permutation group $\langle \mathrm{Aut}(\mathcal{M}), \mathcal{M} \rangle$, so determines $\mathcal{M}$ up to bi-definability. One such condition has been identified by M. Rubin in [12], and it is related to the definability, in $\mathrm{Aut}(\mathcal{M})$, of point stabilisers. If the condition holds, the structure is said to have a *weak* $\forall\exists$ *interpretation*, and $\mathrm{Aut}(\mathcal{M})$ determines $\mathcal{M}$ up to bi-interpretability or, in some cases, up to bi-definability.

A better-known approach to reconstruction is via the 'small index property': an ω-categorical stucture $\mathcal{M}$ has the *small index property* if any subgroup of $\mathrm{Aut}(\mathcal{M})$ of index less than $2^{\aleph_0}$ is open. This guarantees that the abstract group structure of $\mathrm{Aut}(\mathcal{M})$ determines the topology, so if $\mathcal{N}$ is ω-categorical with $\mathrm{Aut}(\mathcal{M}) \cong \mathrm{Aut}(\mathcal{N})$ then $\mathcal{M}$ and $\mathcal{N}$ are bi-interpretable. Most of the structures handled in this paper are known to have the small index property. However, in unpublished work, mentioned at the end of this paper, A. Singerman has shown that there is an ω-categorical structure which has a weak $\forall\exists$-interpretation but does not have the small index property – it is one of the well-known examples, identified by Cherlin and Hrushovski, whose automorphism group has an infinite profinite quotient. There are also familiar examples, the random tournament and the universal homogeneous partial order, which are proved in [12] to have a weak $\forall\exists$-interpretation, but for which the small index property is unknown. On the other hand, there are easy examples with the small index property but no weak $\forall\exists$-interpretation (e.g., an equivalence relation with all classes of size two, or indeed, any ω-categorical structure whose automorphism goup has non-trivial centre – see Proposition 1.2.1 of [2]).

Our belief is that the existence of weak $\forall\exists$-interpretations is rather easier to prove than the small index property, and that there are many different (slightly *ad hoc*)

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