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MODULAR TYPES IN SOME SUPERSIMPLE THEORIES

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Abstract. We consider a small supersimple theory with a property (CS) (close to stability). We prove that if in such a theory T there is a type $p \in S(A)$ (where A is finite) with $SU(p) = 1$ and infinitely many extensions over $acl^{eq}(A)$, then in T there is a modular such type. Also, if T is supersimple with (CS) and $p \in S(\emptyset)$ is isolated, $SU(p) = 1$ and p has infinitely many extensions over $acl^{eq}(\emptyset)$, then p is modular.

§1. Introduction. Simple theories, introduced by Shelah [12], attracted more attention in the mid-90's, due to the results of Kim and Pillay [7], and also because of important algebraic examples. A good exposition may be found in [13].

As stressed in [13], simple theories can be an independent object of study in model theory. However thus far often results on simple theories are weak versions of results on stable theories. Responsible for this is the well-behaved notion of forking in simple theories.

Combinatorial geometry entered stable model theory via the papers of Zilber [14], Cherlin, Harrington and Lachlan [3] and many others. Hrushovski gave it a new impulse in [6], where he investigated locally modular regular types. The resulting geometric model theory became the main part of stable model theory.

There were several efforts to investigate analogous geometric phenomena in simple theories. In [5], the notion of a modular simple theory was defined. Wagner [13] introduced and investigated locally modular families of types and locally modular groups in a simple theory. However, while in stable theories the starting point was the discovery, that in many cases regular types are locally modular, in simple theories we lack corresponding existence results.

In stable theories there are basically two paths leading to existence of locally modular types. The first one is via some algebraic properties of forking. For example local modularity of a minimal type in an $\aleph_0$-stable $\aleph_0$-categorical theory was obtained in this way [3, 11]. There was a hope, that this result holds also in an $\aleph_0$-categorical supersimple theory, however Hrushovski gave a counterexample [13]. This counterexample puts in doubt the possibility of proving existence of locally modular types in simple theories via this way.

Another reason for the existence of locally modular types in stable theories is the interaction of forking and the topological structure of the Stone space of types. Here the first example is local modularity of a properly weakly minimal type [1], later generalized to local modularity of so-called meager types [8]. In this paper

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