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CONTIGUITY AND DISTRIBUTIVITY IN THE ENUMERABLE TURING DEGREES — CORRIGENDUM

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§1. Introduction. A computably enumerable Turing degree $\boldsymbol{a}$ is called *contiguous* iff it contains only a single computably enumerable weak truth table degree (Ladner and Sasso [2]). In [1], the authors proved that a nonzero computably enumerable degree $\boldsymbol{a}$ is contiguous iff it is *locally distributive*, that is, for all $\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{c}$ with $\boldsymbol{a}_1 \cup \boldsymbol{a}_2 = \boldsymbol{a}$ and $\boldsymbol{c} \leq \boldsymbol{a}$, there exist $\boldsymbol{c}_i \leq \boldsymbol{a}_i$ with $\boldsymbol{c}_1 \cup \boldsymbol{c}_2 = \boldsymbol{c}$.

To do this we supposed that W was a computably enumerable set and U a computably set with a Turing functional Φ such that $\Phi^W = U$. Then we constructed computably enumerable sets A_0, A_1, and B together with functionals Γ_0, Γ_1, Γ, and Δ so that

$$\Gamma_0^W = A_0 \wedge \Gamma_1^W = A_1 \wedge \Gamma^{A_0 \oplus A_1} = W \wedge \Delta^W = B,$$

and so as to satisfy all the requirements below.

$$R_{\vec{\Psi},\vec{\Xi}} : V_0 = \Psi_0^B \wedge V_1 = \Psi_1^B \wedge B = \Psi^{V_0 \oplus V_1} \wedge \Xi_0^{A_0} = V_0 \wedge \Xi_1^{A_1} = V_1 \rightarrow$$

$$(\exists \text{ wtt } \Lambda)[\Lambda^W = U].$$

That is, we built a degree-theoretical splitting A_0, A_1 of W and a set $B \leq_T W$ such that if we cannot beat all possible degree-theoretical splittings V_0, V_1 of B then we were able to witness the fact that $U \leq_W W$ (via Λ).

After the proof it was observed that the set U of the proof (page 1222, paragraph 4) needed only to be Δ_2^0. It was then claimed that a consequence to the proof was that every contiguous computably enumerable degree was, in fact, *strongly contiguous*, in the sense that all (not necessarily computably enumerable) sets of the degree had the same weak truth table degree.

Andre Nies [3] observed that while the claim that the set U need not be computably enumerable *was* correct, the conclusion that the degree was strongly contiguous did *not* directly follow. All that followed was that *deg*(A) *was contiguous and had the additional property that for all (not necessarily c.e.) sets* $B \leq_T A$*,* $B \leq_{wtt} A$.

In the original paper, several corollaries from the supposed proof that contiguous equated to strongly contiguous were proven. For example, we proved that no contiguous degree is m-topped. (That is, there is a computably enumerable set A in the degree such that for all computably enumerable sets $B \leq_T A$, $B \leq_m A$.)

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