THE JOURNAL OF SYMBOLIC LOGIC
Volume 67, Number 4, Dec. 2002

BOROVIK-POIZAT RANK AND STABILITY

JEFFREY BURDGES* AND GREGORY CHERLIN†

§1. Introduction. Borovik proposed an axiomatic treatment of Morley rank in groups, later modified by Poizat, who showed that in the context of groups the resulting notion of rank provides a characterization of groups of finite Morley rank [2]. (This result makes use of ideas of Lascar, which it encapsulates in a neat way.) These axioms form the basis of the algebraic treatment of groups of finite Morley rank undertaken in [1].

There are, however, ranked structures, i.e., structures on which a Borovik-Poizat rank function is defined, which are not $\aleph_0$-stable [1, p. 376]. In [2, p. 9] Poizat raised the issue of the relationship between this notion of rank and stability theory in the following terms: " ... un *groupe* de Borovik est une structure stable, alors qu'un univers rangé n'a aucune raison de l'être ... " (emphasis added). Nonetheless, we will prove the following:

THEOREM 1.1. *A ranked structure is superstable.*

An example of a non-$\aleph_0$-stable structure with Borovik-Poizat rank 2 is given in [1, p. 376]. Furthermore, it appears that this example can be modified in a straightforward way to give $\aleph_0$-stable structures of Borovik-Poizat rank 2 in which the Morley rank is any countable ordinal (which would refute a claim of [1, p. 373, proof of C.4]). We have not checked the details. This does not leave much room for strenghthenings of our theorem. On the other hand, the proof of Theorem 1.1 does give a finite bound for the heights of certain trees of definable sets related to unsuperstability, as we will see in Section 5.

Since Shelah gave combinatorial criteria both for instability as well as for unsuperstability in a stable context, to prove the theorem we need only show that these criteria are incompatible with the Borovik-Poizat rank axioms. Now the rank axioms apply only to one structure, while Shelah's criteria take their simplest form in a saturated model. There are two ways to bridge this gap. Our first proof worked directly within the model in which the rank function is defined, paying attention in the process to the *uniformity* of various first order definitions. In the proof we give here, we first extract the first order content of the rank axioms, then work with them directly in a saturated model.

Received February 1, 2002; accepted March 3, 2002.
*Supported by NSF Graduate Research Fellowship.
†Supported by NSF Grant DMS 9803417.

0022-4812/02/6704-0017/$1.90