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A UNIFIED COMPLETENESS THEOREM FOR QUANTIFIED MODAL LOGICS

GIOVANNA CORSI

Abstract. A general strategy for proving completeness theorems for quantified modal logics is provided. Starting from free quantified modal logic K, with or without identity, extensions obtained either by adding the principle of universal instantiation or the converse of the Barcan formula or the Barcan formula are considered and proved complete in a uniform way. Completeness theorems are also shown for systems with the extended Barcan rule as well as for some quantified extensions of the modal logic B. The incompleteness of $Q^\circ.B{+}BF$ is also proved.

In this paper we consider all free and classical quantified extensions of the propositional modal logic K obtained by adding either the axioms of identity or the Converse of the Barcan Formula or the Barcan Formula or the Extended Barcan Rule. Quantified extensions of the propositional logic B are also examined.[1] The lack of " ... a common completeness proof that can cover constant domains, varying domains, and models meeting other conditions ... " has often been felt, see [3], p. 132. In [4] and [5], p. 273, we read "Ideally, we would like to find a completely general completeness proof." The production of such a proof is the aim of this paper. We proceed by presenting a completeness proof for the system $Q^\circ.K$, Kripke's original one[2] with the addition of individual constants, we then show that such a proof yields completeness results for extensions of $Q^\circ.K$ such as those characterized by models with increasing or constant domains, with or without non-existing objects, with or without identity. Our main goal is to offer a clear framework in which each completeness result considered, old or new, will find its natural place. Sometimes we will follow through the proof of a known result just to show how it fits into our framework. In the first part of the paper we will deal with the systems mentioned in the diagram below:

$Q^\circ.K$

$Q^\circ.K + CBF$

$Q^\circ.K + CBF + BF$

$Q.K$

$Q.K + BF$

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[2]See [7].

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