THE JOURNAL OF SYMBOLIC LOGIC
Volume 67, Number 4, Dec. 2002

IKP AND FRIENDS

ROBERT S. LUBARSKY

§1. Introduction. There has been increasing interest in intuitionistic methods over the years. Still, there has been relatively little work on intuitionistic set theory, and most of that has been on intuitionistic ZF. This investigation is about intuitionistic admissibility and theories of similar strength.

There are several more particular goals for this paper. One is just to get some more Kripke models of various set theories out there. Those papers that have dealt with IZF usually were more proof-theoretic in nature, and did not provide models. Furthermore, the inspirations for many of the constructions here are classical forcing arguments. Although the correspondence between the forcing and the Kripke constructions is not made tight, the relationship between these two methods is of interest (see [6] for instance) and some examples, even if only suggestive, should help us better understand the relationship between forcing and Kripke constructions. Along different lines, the subject of least and greatest fixed points of inductive definitions, while of interest to computer scientists, has yet to be studied constructively, and probably holds some surprises. Admissibility is of course the proper set-theoretic context for this study. Finally, while most of the classical material referred to here has long been standard, some of it has not been well codified and may even be unknown, so along the way we'll even fill in a gap in the classical literature.

The next section develops the basics of IKP, including some remarks on fixed points of inductive definitions. After that some classical theories related to KP are presented, and the question of which imply which others is completely characterized. While they are not equivalent in general, when restricted to initial segments of (classical) L they are. However, the section after that shows that in intuitionistic L this equivalence breaks down. We close with some questions.

§2. IKP. The axioms of classical KP are: Empty Set, Pairing, Union, Extensionality, Foundation (as a schema for all definable classes), Δ_0 Comprehension (also known as Δ_0 Separation), and Δ_0 Bounding (also known as Δ_0 Collection). Often Infinity is adjoined; we will also use the axiomatization with Infinity in this paper. There is not much trouble adapting these to an intuitionistic setting. The

Received May 8, 1998; revised November 6, 1998; accepted December 15, 2001.
I would like to thank Anil Nerode for bringing the subject of intuitionistic set theory to my attention, Michael Rathjen for his encouragement in bringing this work to completion, and the referee for the careful reading and insistence on including the details, even when I didn't think they were necessary.

0022-4812/02/6704-0004/$3.80