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AUTOMORPHISM GROUPS OF MODELS OF PEANO ARITHMETIC

JAMES H. SCHMERL

Which groups are isomorphic to automorphism groups of models of Peano Arithmetic? It will be shown here that any group that has half a chance of being isomorphic to the automorphism group of some model of Peano Arithmetic actually is.

For any structure $\mathfrak{A}$, let $\text{Aut}(\mathfrak{A})$ be its automorphism group. There are groups which are not isomorphic to any model $\mathscr{N} = (N, +, \cdot, 0, 1, \leq)$ of PA. For example, it is clear that $\text{Aut}(\mathscr{N})$, being a subgroup of $\text{Aut}((N, <))$, must be torsion-free. However, as will be proved in this paper, *if* $(A, <)$ *is a linearly ordered set and* G *is a subgroup of* $Aut((A, <))$, *then there are models* $\mathscr{N}$ *of* PA *such that* $Aut(\mathscr{N}) \cong G$.

If $\mathfrak{A}$ is a structure, then its automorphism group can be considered as a topological group by letting the stabilizers of finite subsets of A be the basic open subgroups. If $\mathfrak{A}'$ is an expansion of $\mathfrak{A}$, then $\text{Aut}(\mathfrak{A}')$ is a closed subgroup of $\text{Aut}(\mathfrak{A})$. Conversely, for any closed subgroup $G \leq \text{Aut}(\mathfrak{A})$ there is an expansion $\mathfrak{A}'$ of $\mathfrak{A}$ such that $\text{Aut}(\mathfrak{A}') = G$. Thus, if $\mathscr{N}$ is a model of PA, then $\text{Aut}(\mathscr{N})$ is not only a subgroup of $\text{Aut}((N, <))$, but it is even a *closed* subgroup of $\text{Aut}((N, <))$.

There is a characterization, due to Cohn [2] and to Conrad [3], of those groups G which are isomorphic to closed subgroups of automorphism groups of linearly ordered sets. We say that a linearly ordered group $(G, <)$ is a *right-ordered* group if, whenever $a, b, c \in G$ and $a < b$, then $ac < bc$. A group G is *right-orderable* if $(G, <)$ is right-ordered for some linear ordering $<$ of G. Consult [11] for a comprehensive treatment of right-orderable groups. The following conditions on a group G are all equivalent to one another (as can be found in [2], [3], [11]):

(1) *G is right-orderable*;
(2) *for some linearly ordered set $(A, <)$, G is isomorphic to a subgroup of $Aut((A, <))$*;
(3) *for some linearly ordered set $(A, <)$, G is isomorphic to a closed subgroup of $Aut((A, <))$*;
(4) *there is a linearly ordered structure $\mathfrak{A}$ such that $G \cong Aut(\mathfrak{A})$*;
(5) *there is a linearly ordered structure $\mathfrak{A} = (A, <, R)$, where $R \subseteq A^2$ and $|A| = |G|$, such that $G \cong Aut(\mathfrak{A})$.*

It will be proved here that one more equivalence can be added to this list:

(6) *every model $\mathscr{M}$ of* PA *has an elementary extension $\mathscr{N}$ such that $G \cong Aut(\mathscr{N})$.*

It has been shown by Conrad [3] that the class of right-orderable groups is an elementary class which can be recursively axiomatized by a set of universal sentences.

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