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CORE MODELS WITH MORE WOODIN CARDINALS

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In this paper, we shall prove two theorems involving the construction of core models with infinitely many Woodin cardinals. We assume familiarity with [12], which develops core model theory the one Woodin level, and with [10] and [6], which extend the fine structure theory of [5] to mice having many Woodin cardinals. The most important new problem of a general nature which we must face here concerns the iterability of K^c with respect to uncountable iteration trees.

Our first result is the following theorem, a slightly stronger version of which was proved independently and earlier by Woodin.[1] The theorem settles positively a conjecture of Feng, Magidor, and Woodin [2].

THEOREM. *Let Ω be measurable. Then the following are equivalent*:

(a) *for all posets* $\mathbb{P}, \mathbb{Q} \in V_\Omega$, $L(\mathbb{R})^{V^{\mathbb{P}}} \equiv L(\mathbb{R})^{V^{\mathbb{Q}}}$,
(b) *for every poset* $\mathbb{P} \in V_\Omega$, $V^{\mathbb{P}} \models AD^{L(\mathbb{R})}$,
(c) *for every poset* $\mathbb{P} \in V_\Omega$, $V^{\mathbb{P}} \models$ *there is no uncountable sequence of distinct reals in* $L(\mathbb{R})$,
(d) *there is an Ω-iterable premouse of height Ω which satisfies "there are infinitely many Woodin cardinals"*.

It is an immediate corollary that if every set of reals in $L(\mathbb{R})$ is weakly homogeneous, then $\mathrm{AD}^{L(\mathbb{R})}$ holds.[2] We shall also indicate some extensions of the theorem to pointclasses beyond $L(\mathbb{R})$, and mice with more than ω Woodin cardinals.

Our second result is an improved lower bound on the consistency strength of the failure of the unique branches hypothesis (UBH; cf. [3]) for certain sorts of iteration trees. Recall that an iteration tree $\mathcal{T}$ is called *nonoverlapping* iff whenever E and F are extenders such that E is used before F along some branch of $\mathcal{T}$, then $\mathrm{lh}(E) \leq \mathrm{crit}(F)$. (See [11].) Nonoverlapping trees have special importance because the iteration trees which come up in inner model theory are all linear compositions of nonoverlapping trees.

THEOREM. *If there is a nonoverlapping iteration tree $\mathcal{T}$ on V, having distinct cofinal, wellfounded branches b and c, then there is an inner model with infinitely many Woodin cardinals, and if in addition, $\delta(\mathcal{T}) \in ran(i_b) \cap ran(i_c)$, then there is an inner model with a strong cardinal which is a limit of Woodin cardinals.*

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[1]Woodin proved the theorem under the weaker hypothesis that Ω is inaccessible in the Fall of 1991. We proved the theorem as stated in the Spring of 1992.

[2]The author proved this earlier, in the Fall of 1990, by similar but simpler methods.

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