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TRANSFINITE DEPENDENT CHOICE AND ω-MODEL REFLECTION

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Abstract. In this paper we present some metapredicative subsystems of analysis. We deal with reflection principles, ω-model existence axioms (limit axioms) and axioms asserting the existence of hierarchies. We show several equivalences among the introduced subsystems. In particular we prove the equivalence of Σ^1_1 transfinite dependent choice and Π^1_2 reflection on ω-models of Σ^1_1-DC.

§1. Introduction. The formal system of classical analysis is second order arithmetic with full comprehension principle. It was called classical analysis, since classical mathematical analysis can be formalized in it. Often, subsystems of classical analysis suffice as formal framework for particular parts of mathematical analysis. During the last decades a lot of such subsystems have been isolated and proof-theoretically investigated. The subsystems of analysis introduced in this paper belong to *metapredicative* proof-theory. Metapredicative systems have proof-theoretic ordinals beyond Γ_0 but can still be treated by methods of predicative proof-theory only. Recently, numerous interesting metapredicative systems have been characterized. For previous work in metapredicativity the reader is referred to Jäger [3], Jäger, Kahle, Setzer and Strahm [4], Jäger and Strahm [5, 6], Kahle [7], Rathjen [8] and Strahm [11, 12, 13].

Metapredicative subsystems of analysis are for instance: ATR (proof-theoretic ordinal Γ_{ε_0}, e.g., [5]), $\mathsf{ATR} + \Sigma^1_1\text{-}\mathsf{DC}$, $\mathsf{ATR}_0 + \Sigma^1_1\text{-}\mathsf{DC}$ (proof-theoretic ordinal $\varphi 1 \varepsilon_0 0$, $\varphi 1 \omega 0$ respectively, [5]) and FTR, FTR_0 (proof-theoretic ordinal $\varphi 2 0 \varepsilon_0$, $\varphi 200$ respectively, [12]). We introduce in this paper a lot of subsystems of analysis with proof-theoretic ordinals between $\varphi 200$ and $\varphi \varepsilon_0 00$.

Three concepts are of central importance in this paper: ω-models, reflections and hierarchies. Each subsystem, which we shall introduce, deals with one of these concepts. We shall prove equivalences of some subsystems and determine the proof-theoretic ordinal of some of them. To prove these equivalences, we use the method of "pseudohierarchies" (cf. [10]).

In order to define ω-models within subsystems of analysis we have to formalize the notion of an ω-model. This leads to the notion of countable coded ω-model, cf. e.g., [10]. We say that M satisfies φ or that M is an ω-model of φ iff M reflects φ, i.e., iff φ^M holds. For instance, if Ax_{ACA} is a finite axiomatization of (ACA), then M is an ω-model of ACA iff $(Ax_{\mathsf{ACA}})^M$ holds. In the following we

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