

THE TORSIONFREE PART OF THE ZIEGLER SPECTRUM OF RG
WHEN R IS A DEDEKIND DOMAIN AND G IS A FINITE GROUP

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§1. Introduction. For every ring S with identity, the (right) Ziegler spectrum of S , Zg_S , is the set of (isomorphism classes of) indecomposable pure injective (right) S -modules. The Ziegler topology equips Zg_S with the structure of a topological space. A typical basic open set in this topology is of the form

$$(\varphi/\psi) = \{M \in Zg_S : |\varphi(M) : \varphi(M) \cap \psi(M)| > 1\}$$

where φ and ψ are pp -formulas (with at most one free variable) in the first order language L_S for S -modules; let $[\varphi/\psi]$ denote the closed set $Zg_S - (\varphi/\psi)$. There is an alternative way to introduce the Ziegler topology on Zg_S . For every choice of two f.p. (finitely presented) S -modules A, B and an S -module homomorphism $f : A \rightarrow B$, consider the set (f) of the points N in Zg_S such that some S -homomorphism $h : A \rightarrow N$ does not factor through f . Take (f) as a basic open set. The resulting topology on Zg_S is, again, the Ziegler topology.

The algebraic and model-theoretic relevance of the Ziegler topology is discussed in [Z], [P] and in many subsequent papers, including [P1], [P2] and [P3], for instance. Here we are interested in the Ziegler spectrum Zg_{RG} of a group ring RG , where R is a Dedekind domain of characteristic 0 (for example R could be the ring \mathbb{Z} of integers) and G is a finite group. In particular we deal with the R -torsionfree points of Zg_{RG} .

The main motivation for this is the study of RG -lattices (i.e., finitely generated R -torsionfree RG -modules). Their model theory has been treated in several papers (see [T], for instance). Here we try to understand their role within the spectrum.

The analysis of the R -torsionfree part of Zg_{RG} is developed in § 2. RG -lattices are directly dealt with in § 3.

In § 2. we show that every R -torsionfree point of the Ziegler spectrum of RG either is a simple KG -module, where K denotes the quotient field of R , or is R -reduced and is then a point of the Ziegler spectrum of $\hat{R}_P G$ for some maximal prime ideal P , where \hat{R}_P denotes the completion of R at P . Fix such a prime P . We show that the topology on the R -torsionfree R -reduced points which are \hat{R}_P -modules is the same whether these are considered as points of the spectrum of RG or of $\hat{R}_P G$. We also show that every such point is in the topological closure of the set of such points which are $\hat{R}_P G$ -lattices. Then we investigate how these “ P -patches” fit into the Ziegler spectrum of RG .

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