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ON THE BINDING GROUP IN SIMPLE THEORIES

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Abstract. We show that if p is a real type which is almost internal in a formula φ in a simple theory, then there is a type p' interalgebraic with a finite tuple of realizations of p, which is generated over φ. Moreover, the group of elementary permutations of p' over all realizations of φ is type-definable.

Introduction. In stability theory, in order to study the interaction of a type p (over some set A) with a family Σ of partial types over A, an important notion is *internality*: p is Σ-*internal* if for every realization $a \models p$ there are $B \mathop{\smile\hspace{-0.9em}\mid}_A a$ and realizations $\bar{c}$ of Σ such that $a \in \operatorname{dcl}(AB\bar{c})$. A theorem of Hrushovski states that if p is Σ-internal in a stable theory, then the so-called *binding group*, i.e., the group of permutations of the realizations of p induced by automorphisms of the monster model fixing A and all realizations of Σ, is type-definable, together with its action on p. Apart from structural applications (for instance in Hrushovski's proof that unidimensional stable theories are superstable), this theorem also serves to clarify the rôle of the parameter B used in the definition of internality. Since Poizat has shown that if p is Σ-internal, there is some B (consisting of realizations of p) such that *all* realizations $a \models p$ are definable over B and realizations of Σ (a concept Buechler calls *finite generation*), basically an element σ of the binding group is coded by the pair $(B, \sigma(B))$. For more details, the reader may consult [2, Section 4.4], [4, Section 7.4], and [5, Section 2.e].

In a simple theory, the situation is more complicated: Anand Pillay has constructed examples where finite generation and internality differ (our Examples 2 and 3 below). One way to remedy this situation is to introduce *almost* internality and *almost* generation, where the definable closure is replaced by the algebraic closure (or, more generally in the hyperimaginary context, by the bounded closure). It has been shown by the second author that in a simple theory these two notions do indeed agree: if p is almost Σ-internal, it is almost generated over Σ. However, definability rather than algebraicity is essential in defining the binding group. Our Theorem 6 shows how to obtain, from an almost Σ-internal real type p in a simple theory, a Σ-generated imaginary type p' which is closely related to p.

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