

MODULES WITH FEW TYPES OVER SOME FINITE-DIMENSIONAL ALGEBRAS

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Abstract. Using the description of the Ziegler spectrum we characterise modules with various stability-theoretic properties (ω -stability, superstability, categoricity) over certain classes of finite-dimensional algebras. We also show that, for modules over the algebras we consider, having few types is equivalent to being ω -stable.

§1. Preliminaries. With Ziegler's introduction [28] of his topology on the set of indecomposable pure-injective modules over a ring R and with the many uses made of this topology in that paper it became clear that a great deal of information about the model theory of R -modules is implicit in the explicit description of this topological space, the *Ziegler spectrum*, Zg_R , for a given ring R . Indeed, obtaining this (implicit) model-theoretic information has been one of the motivations for subsequent investigation of Ziegler spectra over a variety of rings.

In practice the description of the spectrum has become an algebraic end in itself and the model-theoretic information which may be derived from such a description usually has been left implicit. In this paper we illustrate how the description of the spectrum may be used to characterise the theories of R -modules which: have few types, are totally transcendental, are \aleph_0 - or \aleph_1 -categorical, are superstable. We do this for two types of finite-dimensional algebras where the description of the Ziegler spectrum has been obtained: the finite-dimensional tame hereditary algebras and the path algebra of the quiver Λ_2 . The complete description of the spectrum of a tame hereditary algebra has used the work of a number of people in model theory and in the representation theory of finite-dimensional algebras and may be found in [16], [24] (also see the exposition in [7]). The path algebra of the quiver with relations Λ_2 turned out to be a counterexample to a conjecture of the first author and its spectrum was described in [5] (and [26] is closely related). We try to make this paper reasonably independent of those references by summarising the descriptions of the spectra of these algebras but, of course, for some definitions and more details one should consult those papers and references therein. For the model-theoretic background the reader may consult [14].

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