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GENERALIZED WEAK PRESENTATIONS

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Abstract. Let K be a computable field. Let $\mathscr{F}$ be a collection of recursive functions over K, possibly including field operations. We investigate the following question. Given an r.e. degree $\mathfrak{a}$, is there an injective map $j : K \longrightarrow \mathbb{N}$ such that $j(K)$ is of degree $\mathfrak{a}$ and all the functions in $\mathscr{F}$ are translated by restrictions of total recursive functions.

§1. Introduction. Presentations were originally introduced to formalize the notion of an algorithm on countable mathematical structures. The object under consideration was mapped into the natural numbers and a function on the object was considered recursive if its translation was recursive on the image. The original definition of presentations, as can be found in [7] and [13], required that all the operations of the structure be translated by total recursive functions and also required the image of the presentation to be recursive. The structures that have such presentations are usually called *recursive*.

On the other hand, there are naturally arising algebraic structures which do not have recursive presentations but are embedded in structures which are recursive. The operations of these non-recursive structures can be represented by restrictions of total recursive functions. Existence of such structures can motivate one to define another class of presentations: *weak presentations*, which do not require the image of the presentation map to be recursive but do require that all the operations associated with the object are interpreted by functions extendible to total recursive ones. We state the definition explicitly for fields.

DEFINITION 1.1. Let K be a countable field. Let $j : K \longrightarrow \mathbb{N}$ be an injective map such that there exist total recursive functions $P_+, P_-, P_\times, P_/ : \mathbb{N}^2 \longrightarrow \mathbb{N}$ with the property that for all $x, y \in K$, $P_+(j(x), j(y)) = j(x + y), P_-(j(x), j(y)) = j(x - y), P_\times(j(x), j(y)) = j(x \times y)$, and if $y \neq 0, P_/(j(x), j(y)) = j(x/y)$. Then j is called a weak presentation of K.

Given such a definition, one can consider the following class of problems: given a recursive object, what kind of weak presentations does such an object have? Solving a problem of this type should illuminate the relationship between the algebraic and logical structures of the object under consideration. The study of weak presentations

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