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DEGREE SPECTRA OF RELATIONS ON COMPUTABLE STRUCTURES IN THE PRESENCE OF Δ_2^0 ISOMORPHISMS

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Abstract. We give some new examples of possible degree spectra of invariant relations on Δ_2^0-categorical computable structures, which demonstrate that such spectra can be fairly complicated. On the other hand, we show that there are nontrivial restrictions on the sets of degrees that can be realized as degree spectra of such relations. In particular, we give a sufficient condition for a relation to have infinite degree spectrum that implies that every invariant computable relation on a Δ_2^0-categorical computable structure is either intrinsically computable or has infinite degree spectrum. This condition also allows us to use the proof of a result of Moses [23] to establish the same result for computable relations on computable linear orderings.

We also place our results in the context of the study of what types of degree-theoretic constructions can be carried out within the degree spectrum of a relation on a computable structure, given some restrictions on the relation or the structure. From this point of view we consider the cases of Δ_2^0-categorical structures, linear orderings, and 1-decidable structures, in the last case using the proof of a result of Ash and Nerode [3] to extend results of Harizanov [14].

§1. Introduction. The study of properties of computable structures has formed an important and fertile branch of computable model theory. (A valuable reference is the handbook [9]. In particular, the introduction and the articles by Ershov and Goncharov [8] and Harizanov [12] give useful overviews, while the articles by Ash [1] and Goncharov [10] cover material related to the topic of this paper. Another relevant survey article is [21].) One direction this study has taken, beginning with the work of Ash and Nerode [3] in the early 1980s, concerns the question of what can be said about the images of an additional relation U on the domain of a computable structure $\mathscr{M}$ (that is, one that is not the interpretation in $\mathscr{M}$ of a relation in the language of $\mathscr{M}$) in different computable copies of $\mathscr{M}$.

For example, if $\mathscr{L}$ is a linear ordering of type ω and S is the successor relation on $\mathscr{L}$ then there is a computable copy of $\mathscr{L}$ in which the image of S is computable, namely ω with its standard ordering. But there are also computable copies of $\mathscr{L}$ in which the images of S are not computable (see, for instance, [5]). In fact, for every computably enumerable (c.e.) degree **a**, we can construct a computable linear ordering of type ω in which the successor relation has degree **a**. On the other hand,

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