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ON ORBITS OF PROMPT AND LOW COMPUTABLY ENUMERABLE SETS

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Abstract. This paper concerns automorphisms of the computably enumerable sets. We prove two results relating semilow sets and prompt degrees via automorphisms, one of which is complementary to a recent result of Downey and Harrington. We also show that the property of effective simplicity is not invariant under automorphism, and that in fact every promptly simple set is automorphic to an effectively simple set. A major technique used in these proofs is a modification of the Harrington-Soare version of the method of Harrington-Soare and Cholak for constructing Δ_3^0 automorphisms; this modification takes advantage of a recent result of Soare on the extension of "restricted" automorphisms to full automorphisms.

§1. Introduction.

1.1. Background. A set $A \subseteq \omega$ is *computably enumerable* (*c. e.*) if its elements can be listed by an effective algorithm. The collection of computably enumerable sets can then be given two different kinds of structure, one based on their computational properties, and one based on their algebraic properties:

(1) The c. e. Turing degrees form an upper semilattice $\mathscr{R}$ under $\leq_T$ (Turing reducibility), and
(2) The c. e. sets form a lattice $\mathscr{E}$ under inclusion.

In 1944, Post [7] first looked at the connection between these structures; in the search for a noncomputable incomplete degree, he defined several new properties of c. e. sets (such as creativity, simplicity, and hyperhypersimplicity) that have turned out to be definable purely in terms of set inclusion. Ever since that time, there has been an ongoing program of examining the relationship between the structures of $\mathscr{R}$ and $\mathscr{E}$.

One fruitful area of research has been the study of automorphisms of $\mathscr{E}$. This began in the 1970s, starting with Soare [10], and being further developed by the work of Maass, Stob, Downey, Harrington, and others. This early work involved the construction of *effective* automorphisms of various sorts. Then in the mid 1990s, Harrington and Soare, and independently Cholak, introduced a powerful new method for constructing automorphisms. This method, described in [5], combines

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