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ON THE RAMSEYAN PROPERTIES OF SOME SPECIAL SUBSETS OF 2^ω AND THEIR ALGEBRAIC SUMS

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Abstract. We prove the following theorems:
1. If $X\subseteq 2^\omega$ is a γ–set and $Y\subseteq 2^\omega$ is a strongly meager set, then $X+Y$ is Ramsey null.
2. If $X\subseteq 2^\omega$ is a γ–set and Y belongs to the class of $\mathscr{E}$ sets, then the algebraic sum $X+Y$ is an $\mathscr{E}$ set as well.
3. Under CH there exists a set $X\in MGR^*$ which is not Ramsey null.

Key words and phrases: Strongly meager sets. Ramsey null sets. (T') sets.

§1. Introduction and terminology. In this paper we continue our investigations of some additive properties of special sets of real numbers (see [4] and [5] for previous results). In particular we show that the algebraic sum of a strongly meager set and a set with a certain combinatorial property, called (T') , is Ramsey null. We also show that it is consistent with the ZFC that additively meager sets are not necessarily Ramsey null. This paper is organized as follows. In Section 2 we deal with the Ramseyan properties of small sets of real numbers. Section 3 contains results related to the introduced earlier (T') sets. We also study their connections with other special sets of real numbers. Finally, in Section 4 we prove that in contrast to (T') sets, additively meager sets do not have to be Ramsey null. We finish this section with the three questions that we haven't been able to answer. Throughout this paper we identify sets of real numbers with subsets of the Cantor space 2^ω. By " + " we mean the usual modulo 2 coordinatewise addition in 2^ω. For an infinite $A\subseteq\omega$, $[A]^\omega$ is the set of all infinite subsets of A. Depending on the context, $a\in[A]^\omega$ is often conflated with its characteristic function. Conversely, an element of the Cantor space 2^ω, say b, is sometimes identified with the set $\{i\in\omega : b(i)=1\}$.

§2. Ramsey null sets. First, let us recall a couple of definitions. By MGR and $\mathscr{N}$ we denote the σ-ideals of meager and Lebesgue measure zero sets, respectively. $\mathscr{E}$ is the σ-ideal generated by closed, measure zero sets.

Suppose that $\mathscr{J}$ is an σ - ideal of subsets of the reals. A set X is $\mathscr{J}$-additive if for every set $F\in\mathscr{J}$, $X+F\in\mathscr{J}$. For example, X is meager additive ($X\in MGR^*$) if for every set $F\in MGR$, $X+F\in MGR$.

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