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SPLITTING PROPERTIES OF n-C.E. ENUMERATION DEGREES

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Abstract. It is proved that if $1 < m < 2p \leq n$ for some integer p then the elementary theories of posets of m-c.e. and n-c.e. e-degrees are distinct. It is proved also that the structures $\langle \mathscr{D}_{2n}, \leq, P \rangle$ and $\langle \mathscr{D}_{2n}, \leq, P \rangle$ are not elementary equivalent where P is the predicate $P(\boldsymbol{a}) =$ "$\boldsymbol{a}$ is a Π^0_1 e-degree".

§1. Introduction. A set A is *enumeration reducible to* a set B (in symbols: $A \leq_e B$), if there is an algorithm for enumerating A given any enumeration of B. Namely (see e.g. [1]), if there exists some computably enumerable set W, such that

$$A = \{x : (\exists u)[\langle x, u \rangle \in W \ \&\ D_u \subseteq B]\}$$

where D_u is the finite set with canonical index u (in the following we will often identify finite sets with their canonical indices). Thus, each c. e. set W can be viewed as an operator (called an *enumeration operator*), associating to each set B, the set A which is obtained from B as above. The degree structure originated by this reducibility is the structure of the *enumeration degrees*. (In the following, we will write e-reducible, e-operator, e-degree for enumeration reducible, enumeration operator, enumeration degree, respectively. We will also denote by $\deg_e(A)$ the e-degree of a set A.)

In this paper we study the structure of the n-c.e. e-degrees (where $n \geq 2$). In fact, for each $n \geq 2$, the n-c.e. e-degrees form an upper semilattice $\mathscr{D}_n$ with least element $\boldsymbol{0}$ (the e-degree of the c.e. sets) and greatest element $\boldsymbol{0}'$ (the e-degree of $\overline{K}$, where K is any creative set). Arslanov, Kalimullin and Sorbi proved (see [2]) that every nonzero n-c.e. e-degree strictly bounds some nonzero 3-c.e. e-degree. Hence, in each $\mathscr{D}_n$ there is no minimal e-degree. Moreover, by Corollary 2, every nonzero n-c.e. e-degree nontrivially splits.

It is known ([3]) that the 2-c.e. e-degrees are isomorphic to the c.e. Turing degrees. Note that by [7] it is the unique example of an isomorphism between $\mathscr{D}_n$ (for some $n \geq 2$) and the m-c.e. Turing degrees (for some $m \geq 1$). By Corollary 1 (see below) there is an elementary difference at the Σ_2-level (in the language with signature $\leq$) between $\mathscr{D}_n$ ($n > 2$) and $\mathscr{D}_2$.

Downey [5] conjectured that for $m \geq 2$ the structures of the m-c.e. Turing degrees are pairwise elementarily equivalent. In the context of the e-degrees it is natural to ask whether the theories of the n-c.e. e-degrees (for $n > 2$) pairwisely coincide. The

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