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A CLASSIFICATION OF INTERSECTION TYPE SYSTEMS

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In honour of Roger Hindley on his 60th birthday.

Abstract. The first system of intersection types, Coppo and Dezani [3], extended simple types to include intersections and added intersection introduction and elimination rules ($(\wedge I)$ and $(\wedge E)$) to the type assignment system. The major advantage of these new types was that they were invariant under β-equality, later work by Barendregt, Coppo and Dezani [1], extended this to include an (η) rule which gave types invariant under $\beta\eta$-reduction.

Urzyczyn proved in [6] that for both these systems it is undecidable whether a given intersection type is empty. Kurata and Takahashi however have shown in [5] that this emptiness problem is decidable for the sytem including (η), but without $(\wedge I)$.

The aim of this paper is to classify intersection type systems lacking some of $(\wedge I)$, $(\wedge E)$ and (η), into equivalence classes according to their strength in typing λ-terms and also according to their strength in possessing inhabitants.

This classification is used in a later paper to extend the above (un)decidability results to two of the five inhabitation-equivalence classes. This later paper also shows that the systems in two more of these classes have decidable inhabitation problems and develops algorithms to find such inhabitants.

§1. The system $\lambda\wedge$ and subsystems.

1.1. DEFINITION (Types).

(i) Type variables $a, b, c, \ldots,$ and ω, the universal type, are types.
(ii) If α and β are types so are $(\alpha \to \beta)$ and $(\alpha \wedge \beta)$.

1.2. DEFINITION (TA (type assignment) statements). If α is a type and M a λ-term, $M : \alpha$ is a TA-statement.

1.3. DEFINITION (TA-judgements). If $\Delta = \{x_1 : \alpha_1, \ldots, x_n : \alpha_n\}$ is a set of TA-statements and $M : \alpha$ is a TA-statement then $\Delta \vdash M : \alpha$ is a TA-judgement.

1.4. DEFINITION (The type assignment system $\mathrm{TA}_\lambda(\wedge, \omega)$ or $\lambda \wedge \omega$).

Axiom scheme (ω) $\qquad \vdash M : \omega$

(Var) $\qquad$ if $x : \alpha \in \Delta$, $\quad \Delta \vdash x : \alpha$

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