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A MODEL-THEORETIC PROOF FOR $P \neq NP$ OVER ALL INFINITE ABELIAN GROUPS

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Abstract. We give a model-theoretic proof of the fact that for all infinite Abelian groups $P \neq NP$ in the sense of binary nondeterminism. This result has been announced 1994 by Christine Gaßner.

Introduction. The result proven in this note was announced in a private communication hold by Christine Gaßner in 1994 at the University of Greifswald. When this note was in preparation, the result appeared also in a preprint of Menard Bourgade concerning the polynomial hierarchy over infinite abelian groups. All proofs known so far are complicated and contain a lot of calculations. We will show here a uniform model-theoretic proof.

Our work is compatible with approaches did independently by Poizat [P] and Hemmerling [H] in order to generalize the framework of Blum, Shub and Smale [BSS], [BCSS].

Problems. Given an infinite abelian group G, we call **input** over G a finite non-empty sequence of elements of G. Let G^∞ be the set of all inputs. A **problem** Π over G is any set of inputs ($\Pi \subset G^\infty$). A G**-machine** is a computation system given by a finite description and able to work out inputs of arbitrary length according to a program. The length of an input is the measure of its (algebraic) complexity. By **polynomial time** we mean that the time of computation has at most a polynomial increment rate in the length of the input.

Nondeterminism. In the **binary** (called also boolean -, ramification -, or simply first kind of -) nondeterminism situations in which the machine can continue the computation in two different ways are allowed. The second kind of nondeterministic machines have **guess** instructions, assigning to some register any value picked up arbitrarily from the group. If one algebraic structure contains at least two elements and possess equality one can simulate any binary nondeterministic machine using a guess nondeterministic one.

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