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UNIQUE DECOMPOSITION IN CLASSIFIABLE THEORIES

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§1. Introduction. By a classifiable theory we shall mean a theory which is superstable, without the dimensional order property, which has prime models over pairs. In order to define what we mean by unique decomposition, we remind the reader of several definitions and results. We adopt the usual conventions of stability theory and work inside a large saturated model of a fixed classifiable theory T; for instance, if we write $M \subseteq N$ for models of T, M and N we are thinking of these models as elementary submodels of this fixed saturated models; so, in particular, M is an elementary submodel of N. Although the results will not depend on it, we will assume that T is countable to ease notation.

We do adopt one piece of notation which is not completely standard: if T is classifiable, $M_0 \subseteq M_i$ for $i = 1, 2$ are models of T and M_1 is independent from M_2 over M_0 then we write $M_1 \oplus_{M_0} M_2$ for the prime model over $M_1 \cup M_2$.

DEFINITION 1.1.

1. If $M \subseteq N$ are models of T then $M \subseteq_{na} N$ if whenever $\varphi(x) \in L(M)$ such that $\varphi(N) \setminus M$ is non-empty and $F \subseteq M$ is any finite set then $\varphi(M) \setminus \operatorname{acl}(F)$ is non-empty.
2. We write $M \subseteq_{\aleph_1} N$ and say that M is a relatively $\aleph_1$-saturated substructure of N if, whenever A and B are countable subsets of M and N respectively, there is B' in M with the same type as B over A.
3. If $M_0 \subseteq M \subseteq N$ then M is an M_0-component of N if the weight of $\operatorname{tp}(M/M_0)$ is 1 and M is maximal with respect to domination over M_0; i.e., if $M \subseteq X \subseteq N$ and M dominates X over M_0 then $M = X$.

The following Theorem from [1] explains the importance of components in classifiable theories.

THEOREM 1.2. *If T is classifiable and N is a model of T with $M \subseteq_{na} N$ then N is prime and minimal over any maximal M-independent collection of M-components of N.*

Keeping the notation of the previous Theorem, we will call such a maximal collection of M-components an M-component decomposition of N or simply a

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