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CORRIGENDUM TO: 'A CONSTRUCTIVE VIEW ON ERGODIC THEOREMS'

BAS SPITTERS

Jeremy Avigad kindly pointed out to me that the proof of Theorem 2 in [3] is incomplete. At the very end it assumes that when $\mathscr{N}^{\perp\perp}$ is located, so is $\mathscr{N}$. This is not correct. In fact, Avigad explained that the construction in the proof of Theorem 15.3 in [1] can be read as a recursive counterexample to the statement: 'If $\mathscr{M} = 0$, then $A_n x$ converges for every x'. As a result the conclusion of Theorem 2 should be that A_n converges if and only if $\mathscr{N}$ is located. Similar changes have to be made in the abstract, Lemma 4, and Theorem 16. Finally, the paragraph on ergodic measure preserving transformations should be removed.

As an unrelated point we would like to mention that in the last two displayed equations of the proof of theorem 1 the term $\frac{1}{n}T^{n-1}$ should be replaced by $\frac{1}{n}T^{n-2}$. The correct statements are as follows:

Abstract. Let T be a positive L_1-L_∞ contraction. We prove that the following statements are equivalent in constructive mathematics.

1. The projection in L_2 on the space $\mathscr{N} := \mathrm{cl}\{x - Tx : x \in L_2\}$ exists;
2. The sequence $(T^n)_{n\in\mathbf{N}}$ Cesàro-converges in the L_2 norm;
3. The sequence $(T^n)_{n\in\mathbf{N}}$ Cesàro-converges almost everywhere.

Thus, we find necessary and sufficient conditions for the Mean Ergodic Theorem and the Dunford-Schwartz Pointwise Ergodic Theorem.

THEOREM 2 (Mean Ergodic Theorem). *Let T be a contraction on a Hilbert space $\mathscr{H}$. Then the sequence $(A_n)_{n\in\mathbf{N}}$ converges if and only if $\mathscr{N}$ is located; in this case the sequence $(A_n)_{n\in\mathbf{N}}$ converges to the orthogonal projection $P_{\mathscr{M}}$ on $\mathscr{M}$.*

We provide a slight strengthening of Theorem 9 in [3]. It allows us to leave the statement of Theorem 10 almost unchanged, only changing $\mathscr{M}$ into $\mathscr{N}$ in the first item below.

THEOREM. *Let $p \geq 1$. If the sequence $(A_n f)_{n\in\mathbf{N}}$ converges a.e. for all f in L_p, then $\mathscr{N} = \mathrm{cl}\{x - Tx : x \in L_2\}$ is located in L_2.*

PROOF. Define $B_n := (\frac{n-1}{n}I + \frac{n-2}{n}T + \cdots + \frac{1}{n}T^{n-2})$. The proof of Theorem 9 in [3] shows that for each x, $(I - T)B_n x$ converges weakly to an element on the space $\mathrm{wcl}\{y - Ty : y \in L_2\}$. Since $(I - T)B_n x = (I - A_n)x$ is a bounded sequence and the weak closure of a bounded convex inhabited subset coincides with its strong closure, by Lemma 5.2.4 in [2], we see that $\mathscr{N} = \mathrm{wcl}\{y - Ty : y \in L_2\}$. Since the

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