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FINITE SATISFIABILITY AND $\aleph_0$-CATEGORICAL STRUCTURES WITH TRIVIAL DEPENDENCE

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Introduction. The main subject of the article is the finite submodel property for $\aleph_0$-categorical structures, in particular under the additional assumptions that the structure is simple, 1-based and has trivial dependence. Here, a structure has the finite submodel property if every sentence which is true in the structure is true in a finite substructure of it. It will be useful to consider a couple of other finiteness properties, related to the finite submodel property, which are variants of the usual concept of saturation.

For the rest of the introduction we will assume that M is an $\aleph_0$-categorical (infinite) structure with a countable language. We also assume that there is an upper bound to the arity of the function symbols in M:s language and that, for every $0 < n < \aleph_0$ and $R \subseteq M^n$ which is definable in M without parameters, there exists a relation symbol, in the language of M, which is interpreted as R; these assumptions are not necessary for most results to be presented, but it simplifies the statement of a result which I mention in this introduction.

First we will consider 'canonically embedded' substructures of M^{eq}. Here, a structure N is canonically embedded in M^{eq} if N's universe is a subset of M^{eq} which is definable without parameters and, for every $0 < n < \aleph_0$ and $R \subseteq N^n$ which is $\emptyset$-definable in M^{eq} there is a relation symbol in the language of N which is interpreted as R; we also assume that the language of N has no other relation (or function or constant) symbols. We prove that if $N \subseteq M^{\mathrm{eq}}$ is a structure which is canonically embedded in M^{eq}, only finitely many sorts are represented in N and M is included in the algebraic closure of N (where algebraic closure is taken in M^{eq}), then M has the finite submodel property if and only if N has it; except for the assumptions on the language of M we only assumed that M is $\aleph_0$-categorical.

Then, in Section 3, we show that, under the additional assumptions that M is simple, 1-based and has trivial dependence (which implies that M has finite SU-rank), there exists a structure $N \subseteq M^{\mathrm{eq}}$ such that N is canonically embedded in M^{eq}, only finitely many sorts are represented in N, M is included in the algebraic closure of N and the algebraic closure restricted to N forms a trivial (or degenerate) pregeometry.

Let N be as above and let acl_N denote the algebraic closure in N (which is the same as the algebraic closure in M^{eq} restricted to N), so (N, acl_N) is a pregeometry. Then, to N we may apply results from [4] where $\aleph_0$-categorical structures M' such

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