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AUTOMORPHISM GROUPS OF ARITHMETICALLY SATURATED MODELS

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§0. Introduction. In this paper we study the automorphism groups of countable arithmetically saturated models of Peano Arithmetic. The automorphism groups of such structures form a rich class of permutation groups. When studying the automorphism group of a model, one is interested to what extent a model is recoverable from its automorphism group. Kossak-Schmerl [12] show that if M is a countable, arithmetically saturated model of Peano Arithmetic, then $\mathrm{Aut}(M)$ codes $\mathrm{SSy}(M)$. Using that result they prove:

THEOREM 0.1. *Let M_1, M_2 be countable arithmetically saturated models of Peano Arithmetic such that* $\mathrm{Aut}(M_1) \cong \mathrm{Aut}(M_2)$. *Then* $\mathrm{SSy}(M_1) = \mathrm{SSy}(M_2)$.

We show that if M is a countable arithmetically saturated of Peano Arithmetic, then $\mathrm{Aut}(M)$ can recognize if some maximal open subgroup is a stabilizer of a nonstandard element, which is smaller than any nonstandard definable element. That fact is used to show the main theorem:

THEOREM 0.2. *Let M_1, M_2 be countable arithmetically saturated models of Peano Arithmetic such that* $\mathrm{Aut}(M_1) \cong \mathrm{Aut}(M_2)$. *Then for every $n < \omega$*

$$(\omega, \mathrm{Rep}(\mathrm{Th}(M_1))) \models \mathrm{RT}_2^n \text{ iff } (\omega, \mathrm{Rep}(\mathrm{Th}(M_2)) \models \mathrm{RT}_2^n .$$

Here RT_2^n is Infinite Ramsey's Theorem stating that every 2-coloring of $[\omega]^n$ has an infinite homogeneous set. Theorem 0.2 shows that for models of a false arithmetic the converse of Kossak-Schmerl Theorem 0.1 is not true. Using the results of Reverse Mathematics we obtain the following corollary:

COROLLARY 0.3. *There exist four countable arithmetically saturated models of Peano Arithmetic such that they have the same standard system but their automorphism groups are pairwise non-isomorphic.*

The corollary is an improvement of a previous result [8] which shows the existence of only 2 such models.

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