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BI-BOREL REDUCIBILITY OF ESSENTIALLY COUNTABLE BOREL EQUIVALENCE RELATIONS

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This note answers a questions from [2] by showing that considered up to Borel reducibility, there are more essentially countable Borel equivalence relations than countable Borel equivalence relations. Namely:

THEOREM 0.1. *There is an essentially countable Borel equivalence relation E such that for* no *countable Borel equivalence relation F (on a standard Borel space) do we have*

$$E \sim_B F.$$

The proof of the result is short. It does however require an extensive rear guard campaign to extract from the techniques of [1] the following

MESSY FACT 0.2. *There are countable Borel equivalence relations $(E_x)_{x\in 2^{\mathbb{N}}}$ such that:*

(i) *each E_x is defined on a standard Borel probability space (X_x, μ_x); each E_x is μ_x-invariant and μ_x-ergodic;*
(ii) *for $x_1 \neq x_2$ and A μ_{x_1}-conull, we have $E_{x_1}|_A$ not Borel reducible to E_{x_2};*
(iii) *if $f : X_x \to X_x$ is a measurable reduction of E_x to itself, then $\mu_x(\mathrm{im}(f)) > 0$;*
(iv)

$$\bigcup_{x\in 2^{\mathbb{N}}} \{x\} \times X_x$$

is a standard Borel space on which the projection function

$$(x, z) \mapsto x$$

is Borel and the equivalence relation $\hat{E}$ given by

$$(x, z)\hat{E}(x', z')$$

if and only if $x = x'$ and $zE_x z'$ is Borel;

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