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SUBSETS OF SUPERSTABLE STRUCTURES ARE WEAKLY BENIGN

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Baizhanov and Baldwin [1] introduce the notions of benign and weakly benign sets to investigate the preservation of stability by naming arbitrary subsets of a stable structure. They connect the notion with work of Baldwin, Benedikt, Bouscaren, Casanovas, Poizat, and Ziegler. Stimulated by [1], we investigate here the existence of benign or weakly benign sets.

Definition 0.1. (1) *The set A is* benign *in M if for every α, $\beta \in M$ if $p = \mathrm{tp}(\alpha/A) = \mathrm{tp}(\beta/A)$ then $\mathrm{tp}_*(\alpha/A) = \mathrm{tp}_*(\beta/A)$ where the $*$-type is the type in the language L^* with a new predicate P denoting A.*

(2) *The set A is* weakly benign *in M if for every $\alpha, \beta \in M$ if $p = \mathrm{stp}(\alpha/A) = \mathrm{stp}(\beta/A)$ then $\mathrm{tp}_*(\alpha/A) = \mathrm{tp}_*(\beta/A)$ where the $*$-type is the type in language with a new predicate P denoting A.*

Conjecture 0.2 (too optimistic). *If M is a model of stable theory T and $A \subseteq M$ then A is benign.*

Shelah observed, after learning of the Baizhanov-Baldwin reductions of the problem to equivalence relations, the following counterexample.

Lemma 0.3. *There is an ω-stable rank 2 theory T with ndop which has a model M and set A such that A is not benign in M.*

Proof. The universe of M is partitioned into two sets denoted by Q and R. Let Q denote $\omega \times \omega$ and R denote $\{0, 1\}$. Define $E(x, y, 0)$ to hold if the first coordinates of x and y are the same and $E(x, y, 1)$ to hold if the second coordinates of x and y are the same. Let A consist of one element from each $E(x, y, 0)$-class and one element of all but one $E(x, y, 1)$-class such that no two members of A are equivalent for either equivalence relation. It is easy to check that letting α and β denote the two elements of R, we have a counterexample. In this case, the type p is algebraic. Algebraicity is a completely artificial restriction. Replace each α and β by an infinite set of points which behave exactly as α, β respectively. We still have a counterexample. In either case, α and β have different strong types. This leads to the following weakening of the conjecture. ⊣

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