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MODEL COMPLETENESS OF O-MINIMAL STRUCTURES EXPANDED BY DEDEKIND CUTS

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§1. Introduction. Let M be a totally ordered set. A (Dedekind) cut p of M is a couple (p^L, p^R) of subsets p^L, p^R of M such that $p^L \cup p^R = M$ and $p^L < p^R$, i.e., $a < b$ for all $a \in p^L,\ b \in p^R$. In this article we are looking for model completeness results of o-minimal structures M expanded by a set p^L for a cut p of M. This means the following. Let M be an o-minimal structure in the language $\mathscr{L}$ and suppose M is model complete. Let $\mathscr{D}$ be a new unary predicate and let p be a cut of (the underlying ordered set of) M. Then we are looking for a natural, definable expansion of the $\mathscr{L}(\mathscr{D})$-structure (M, p^L) which is model complete.

The first result in this direction is a theorem of Cherlin and Dickmann (cf. [Ch-Dic]) which says that a real closed field expanded by a convex valuation ring has a model complete theory. This statement translates into the cuts language as follows. If Z is a subset of an ordered set M we write Z^+ for the cut p with $p^R = \{a \in M \mid a > Z\}$ and Z^- for the cut q with $q^L = \{a \in M \mid a < Z\}$. We call Z^+ the **upper edge** of Z and Z^- the **lower edge** of Z. Then the Cherlin-Dickmann theorem says that (M, p^L) is model complete if p is the upper edge of a convex valuation ring of a real closed field M. This theorem has been generalized by van den Dries and Lewenberg in [vdD-Lew], for o-minimal expansions M of real closed fields and so called T-convex subrings of M (where T is the theory of M; a T-convex valuation ring is the convex hull of an elementary restriction of M, cf. 4.3).

If p is not an edge of a convex valuation ring of a real closed field M then one can show that the $\mathscr{L}(\mathscr{D})$-theory (where $\mathscr{L}$ is the language of ordered rings) of (M, p^L) is not model complete (cf. [T1], §16). So for model completeness we actually have to extend the language $\mathscr{L}(\mathscr{D})$.

We do the following. Let M be again an o-minimal expansion of a real closed field and let p be a cut of M. Let $G(p) := \{a \in M \mid a + p = p\}$, where $a + p := (a + p^L, a + p^R)$. $G(p)$ is a convex subgroup of $(M, +, \leq)$, called the invariance group of p. Moreover $V(p) := \{b \in M \mid b \cdot G(p) \subseteq G(p)\}$ is a convex valuation ring of M, called the invariance ring of p. Suppose there is a T-convex valuation ring V of M, such that $V(p)$ is definable in (M, V) (this is no restriction if M is a pure real closed field). Then our results 3.8 and 7.4 imply that we get model completeness of $Th(M, p^L)$ if we expand the language $\mathscr{L}(\mathscr{D})$ by a unary predicate

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